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Elitist Continuous Ant Colony Optimization Algorithm for Optimal Management of Coastal Aquifers

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Abstract This paper presents an evolutionary based approach to achieve optimal management of a coastal aquifer to control saltwater intrusion. An improved Elitist Continuous Ant Colony Optimization (ECACO) algorithm is employed for optimal control variables setting of coastal aquifer management problem. The objectives of the optimal management are; maximizing the total water-pumping rate, while controlling the drawdown limits and protecting the wells from saltwater intrusion. Since present work is one of the first efforts towards the application of an ECACO algorithm, sharp interface solution for steady state problem is first exploited. The performance of the developed optimization model is evaluated through application examples available in the literature. The comparisons indicate the applicability of the ECACO algorithm. In the second approach, the numerical simulation is combined with ECACO algorithm. In this model, through some simple schemes, such as continuity equations in the porous media cells and existing hydraulic systems in the study area, further details can be investigated. The evaluation results show the potential applicability of the proposed numerical based model for optimal management of coastal aquifers.

Keywords Simulation · Coastal aquifer · Continuous ant colony optimization (CACO) · Optimal management · Saltwater intrusion

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1 Introduction

Groundwater aquifers are important resources in the coastal zones. In coastal areas that are densely populated or have been industrialized, often, serious environmental problems occur. Contaminants and nutrients which are carried by groundwater flow and discharged into coastal waters have a considerable influence on environmental management. Common problems include saltwater intrusion due to over-pumping of groundwater and brine discharges from desalination plants, as well as coastal water pollution by plume leachate from contaminated coastal aquifers (Bear 1979; Ataie-Ashtiani et al. 2002; Ataie-Ashtiani 2007). The exploitation of coastal aquifers is often limited due to excessive saltwater intrusion. Efficient management strategies are needed for optimal exploitation of water from coastal aquifers, while maintaining salt concentration under specified acceptable limit and simultaneously, meeting the required demands.

The development of any sufficient meaningful management strategies requires development of management models that integrate the simulation of the saltwater intrusion process. Simulation of the saltwater intrusion into coastal aquifers can be a complex task. The location, shape and extent of the saltwater dispersion zone depend upon several factors including the scale, the shape or the structure of the aquifer, and the parameters such as hydraulic conductivity. Other influential factors are groundwater inflow from the upstream region, as well as groundwater withdrawals through pumping, and the tidal oscillation of the sea level (Ataie-Ashtiani et al. 1999a, b, 2001).

Reilly and Goodman (1985) presented an historical perspective of saltwater intrusion in coastal aquifers. Saltwater intrusion problems have been solved using different methods, ranging from the basic Badon–Ghyben–Herzberg principle with the sharp interface models to the more sophisticated theories with the solute transport models that allow for variable densities (Bear 1979; Ataie-Ashtiani et al. 2002; Ataie-Ashtiani 2007). Table 1 summarized some studies concerning saltwater intrusion in the coastal aquifer management problems based on sharp interface and density dependent flow assumptions. The focus of this study is on the approaches for achieving the optimal management strategies of coastal aquifer to control saltwater intrusion. A simplified analytical and also, a numerical solution for saltwater intrusion is implemented in this approach.

Optimization methods have been used in many areas of engineering, business, and sciences for decision-making problems (Karterakis et al. 2007). In the field of optimal management of coastal aquifers with saltwater intrusion vulnerability, previous works have obtained optimal solutions of groundwater management problems at varying degrees of success, with the application of linear, nonlinear or heuristic optimization algorithms in conjunction with simulation models (Gorelick and Voss 1984; Shamir et al. 1984; Finney et al. 1992; Karatzas and Pinder 1993; Rizzo and Dougherty 1996; Rogers et al. 1995; Hallaji and Yazicigil 1996; Das and Datta 1999; Cheng et al. 2000; Gordon et al. 2000; Cai et al. 2001; Mantoglou 2003; Mantoglou et al. 2004; Park and Aral 2004; Qahman et al. 2005; Karterakis et al. 2007; Mantoglou and Papantoniou 2008; Dhar and Datta 2009a, b). Depending on the conditions, for proposed management problem, various different objective functions and sets of constraints have been applied. Shamir et al. (1984), Hallaji and Yazicigil

Table 1 Summary of some the studies concerning saltwater intrusion in the coastal aquifer

References	Simulation model	Management model	Application
Sharp interface modeling approach			
Shamir et al. (1984)	2D FDM	LP (MPSX/370)	Regional aquifer
Willis and Finney (1988)	3D FDM	NLP	Yun Lin groundwater basin in Taiwan
Finney et al. (1992)	3D FDM (SHARP)	NLP (MINOS)	Jakarta coastal aquifer in Indonesia
Emch and Yeh (1998)	3D FDM (SHARP)	NLP (MINOS)	Hypothetical aquifer
Cheng et al. (2000)	Analytical	GA	Hypothetical aquifer
Mantoglou (2003)	Analytical	LP and NLP	Hypothetical aquifer
Mantoglou et al. (2004)	Analytical/FDM (MODFLOW)	NLP and EA	Hypothetical aquifer and Greek island of Kalymnos aquifer
Park and Aral (2004)	Analytical	GA	Hypothetical aquifer
Katsifarakis and Petala (2006)	BEM	GA	Hypothetical aquifer
Ferreira da Silva and Haie (2007)	Analytical	EA	Hypothetical aquifer
Uddameri and Kuchanur (2007)	3D FEM (MODFLOW)	LP	Refugio County, TX
Karterakis et al. (2007)	Analytical	LP and DE	Coastal karstic aquifer in Crere, Greece
Mantoglou and Papantoniou (2008)	Analytical/FDM (MODFLOW)	GA and NLP	Hypothetical aquifer and Greek island of Kalymnos aquifer
Density dependent modeling approach			
Gorelick and Voss (1984)	2D FEM/FDM (SUTRA)	NLP (MINOS)	Hypothetical aquifer
Ataie-Ashtiani et al. (1999b)	2D FEM/FDM (SUTRA)	Sensitivity analyses	Hypothetical aquifer
Das and Datta (1999)	3D FDM	NLP (MINOS)	Hypothetical aquifer
Das and Datta (2000)	3D FDM	NLP (MINOS)	Hypothetical aquifer
Gordon et al. (2000)	2D FEM	NLP (QL0001)	Hypothetical aquifer
Bhattacharjya and Datta (2005)	3D FDM (ANN)	GA	Hypothetical aquifer
Qahman et al. (2005)	3D FEM/FDM (CODESA3D)	GA	Hypothetical aquifer
Bhattacharjya et al. (2007)	3D FEM (FEMWATER/ANN)	Sensitivity analyses	Hypothetical aquifer
Kourakos and Mantoglou (2009)	3D FDM (SEAWAT)	EASS	Greek island of Santorini
Dhar and Datta (2009a)	3D FEM (FEMWATER/ANN)	GA (NSGA-II)	Hypothetical aquifer
Dhar and Datta (2009b)	3D FEM (FEMWATER/ANN)	GA (NSGA-II)	Hypothetical aquifer

LP linear programming, *NLP* non-linear programming, *EA* evolutionary algorithm, *GA* genetic algorithm, *DE* differential evolution, *ANN* artificial neural network, *BEM* boundary element method, *FDM* finite difference method, *FEM* finite element method, *EASS* evolutionary annealing simplex scheme optimization algorithm

(1996), Cheng et al. (2000), Mantoglou et al. (2004), and Park and Aral (2004) tried to maximize the total pumping rate, while Das and Datta (1999) planned to minimize the salinity of pumped water from wells. Emch and Yeh (1998), Katsifarakis et al. (1999), and Gordon et al. (2000) presented a model including the pumping cost in the objective function. It is also possible to consider multiple objectives, representing a multi-objective optimization problem (Shamir et al. 1984; Willis and Finney 1988; Emch and Yeh 1998; Das and Datta 1999; Park and Aral 2004).

Coastal aquifer management problems are typically nonlinear and non-convex mathematical programming problems (Ataie-Ashtiani et al. 1999a, b, 2001; Shamir et al. 1984). For such problems, use of classical gradient-based optimization algorithms may result in some unexpected situations. These algorithms usually require good initial solutions to produce an optimal solution. Also, they rely on local gradients of the objective function to determine the search direction, and thus, may converge to local optimal solutions (Cheng et al. 2000; Mantoglou et al. 2004; Park and Aral 2004; Karterakis et al. 2007). For the past three decades, however, many new algorithms have been developed, resulting to efficient optimization algorithms such as the Evolutionary Algorithms (EAs). In contrast to other adaptive algorithms, evolutionary techniques work on a set of potential solutions, which is called, population, and find the optimal solution through cooperation and competition among the potential solutions. These techniques can easily find optimal solutions of the complicated optimization problems in comparison with traditional optimization methods. The most commonly used population-based evolutionary computation techniques, such as Genetic Algorithms (GAs) and artificial life methods such as Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), etc. are motivated from evolution of nature and social behavior (Afshar 2005, 2007; Karterakis et al. 2007; Dhar and Datta 2009a, b).

The applied simplified approach to simulate the saltwater intrusion in the rectangular coastal aquifer in combination with different configurations of GAs in the optimization process has been used by several researchers (Cheng et al. 2000; Mantoglou 2003; Mantoglou et al. 2004; Park and Aral 2004; Katsifarakis and Petala 2006; Mantoglou and Papantoniou 2008; Dhar and Datta 2009a, b). Generally, the purpose of optimization is to maximize the total pumping rate from a number of wells and determination of an optimal pumping scheme while controlling the saltwater intrusion into the coastal aquifer. The constraints control the pumping rate between a minimum and a maximum value (Emch and Yeh 1998; Hallaji and Yazicigil 1996; Das and Datta 1999; Cheng et al. 2000; Park and Aral 2004; Katsifarakis and Petala 2006). Constraints imposed by Cheng et al. (2000), Mantoglou (2003), Mantoglou et al. (2004), Park and Aral (2004), Ferreira da Silva and Haie (2007), Mantoglou and Papantoniou (2008) managed the location of the toe, with regards to the critical stagnation points of existing wells. Further constraints may consist of sustaining water levels, flow potential or salt concentration of the pumped water at desired levels, desired energy use for pumping or recharge, etc.

Cheng et al. (2000) combined a Genetic Algorithm (GA) with sharp interface saltwater intrusion model with the purpose of maximize the total exploited fresh groundwater without observation of saltwater in the wells. The imposed constraints were added in the objective function as penalty terms and the GA was used to solve the new unconstrained problem. The performance of the developed optimization model was evaluated using three elementary application examples, presented for the first time. The obtained results showed the potential applicability of the developed methodology using a GA-based linked optimization–simulation model for optimal management of coastal aquifer. Mantoglou (2003), Mantoglou et al. (2004) and Mantoglou and Papantoniou (2008) investigated a saltwater intrusion approach where the constraints are expressed as analytical functions in the optimization methodology to obtain the optimal pumping strategy. The methods of assessing the optimum pumping rates of coastal aquifers based on linear, nonlinear optimization (simplex method and sequential quadratic programming, respectively), and EAs were developed. The management model was noted to be very sensitive to the aquifer's heterogeneities and recharge rates. The simulation and optimization methodology was applied to a real unconfined coastal aquifer in the Greek island of Kalymnos. Park and Aral (2004) developed a multi-objective GA-based optimization approach to determine the optimal planning and operating policies of a coastal aquifer threatened by saltwater intrusion. Pumping rates and well locations were verified in this approach, while satisfying desired extraction rates in coastal aquifers and no observed saltwater intrusion in the wells. The proposed method was an iterative sub-domain method, which the algorithm searched for the optimal solutions as continuous independent variables, by perturbing the well locations and pumping rates simultaneously. The multi-objective problem was formulated to maximize pumping rates while minimizing the distance between critical stagnation point and the reference coastline location, such that the wells were placed as closely to the coast as possible. As a case study, the numerical results obtained from the proposed method were compared with the work of Cheng et al. (2000). The evaluations showed that the proposed approach provided a cost effective solution to an important management problem in coastal aquifers.

This paper presents an efficient method for optimal management of coastal aquifers with saltwater intrusion, by integrating the improved Elitist Continuous Ant Colony Optimization (ECACO) algorithm with an analytical steady state sharp interface simulation (the first approach) and numerical sharp interface simulation (the second approach) in the assessment of the extent of saltwater intrusion.

More recently, there has been a considerable increase of using ACO algorithms for water resources planning, management, and design. The examples of the use of ACO approach in the solution of water management can be found in the literature such as; reservoir operation (Afshar et al. 2006), water distribution systems (Afshar 2007; Zecchin et al. 2007), groundwater monitoring design (Amy and Hilton 2007), hydraulic parameters of soil estimation (Abbaspour et al. 2009). However, an extremely limited number of researchers have attempted to investigate the coastal aquifer management problems including saltwater intrusion using ACO technique as an optimization tool. The present study investigates the application of ACO approach performance to optimal solution process in combination with analytical and numerical solution of saltwater intrusion in the coastal aquifers. In the numerical simulation, further details can be explored. To demonstrate the efficiency of these integrated techniques, the proposed tools are illustrated through application examples reported by Cheng et al. (2000) and Park and Aral (2004).

2 Continuous Ant Colony Optimization (CACO) Algorithm

Swarm intelligence is a relatively new approach to problem solving that takes inspiration from the social behaviors of insects and of other animals. In particular, ants have inspired a number of methods and techniques among which the most studied and the most successful is the general-purpose optimization technique known as Ant Colony Optimization (ACO) in the class of meta-heuristics (Dorigo and Stützle 2004; Dorigo and Socha 2007). ACO takes inspiration from the foraging behavior of some ant species. These ants deposit a chemical substance called pheromone on the ground in order to mark some favorable path that should be followed by other members of the colony. ACO exploits a similar mechanism for solving optimization problems and maintains a population of cooperate artificial ants, which examine a set of feasible solutions, using pheromone-mediated communication to allocate information. The first ACO system was introduced by Dorigo, and called Ant System (AS) (Dorigo and Stützle 2004). This method was initially proposed for solving combinatorial optimization problems such as the Traveling Salesman Problem (TSP), Quadratic Assignment Problem (QAP), vehicle routing problem, scheduling, timetabling, graph coloring (Afshar et al. 2006; Afshar 2007; Amy and Hilton 2007).

Since the appearance of ant algorithms as an optimization tool, some efforts were also made to use them for attempting continuous optimization problems, particularly in engineering. However, at the first view, applying the ACO meta-heuristic to continuous domain was not straightforward. Hence, the methods proposed often represented inspiration from ACO, but did not follow exactly the same methodology (Afshar et al. 2006; Socha and Dorigo 2008). Early applications of ACO algorithms to continuous optimization include algorithms such as Continuous ACO (CACO), Asynchronous Parallel Implementation (API), Continuous Interacting Ant Colony (CIAC), and Elitist Continuous ACO (ECACO) (Afshar et al. 2006; Madadgar and Afshar 2009). However, all these approaches are conceptually different from ACO for discrete problems. The latest approach, considering in the present study, is closest to the spirit of ACO for discrete problems.

Figure 1 illustrates a flowchart of the improved ECACO algorithm that is appropriate in the present optimization procedure. In ACO algorithms for discrete optimization problems, solutions are constructed by sampling at each construction step a discrete probability distribution that is resulting from the pheromone information. In a way, the pheromone information characterizes the stored search experience of the algorithm. The choice of a solution component from is done probabilistically at each construction step. The exact rules for probabilistic choice of solution components vary across different variants of ACO. The logical adaptation would be to move from using the discrete probability distribution to a continuous one, the Probability Density Function (PDF). Different methods for PDF use can be suggested for calculating pheromone information (Afshar et al. 2006; Socha and Dorigo 2008). Improved ECACO algorithm permits only the ant which has produced the best solution of the iteration to put into pheromone information. Proposed PDF has the main advantage of easily generating random numbers; however, it also has some disadvantages. A single normal PDF is not able to describe a situation where two disjoint areas of the search space are promising, as it only has one optimum.

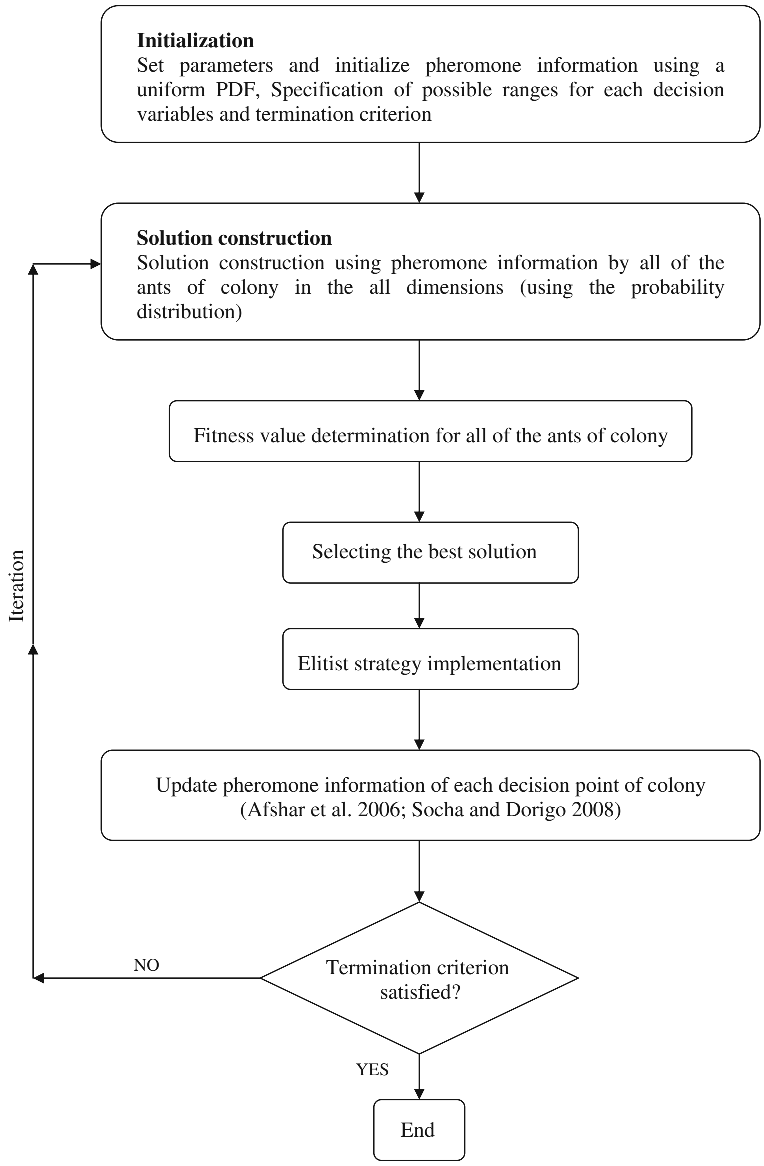


Fig. 1 A flowchart of the elitist continuous ant colony optimization (ECACO) algorithm

This normal distribution for calculating pheromone information is defined as follows (Afshar et al. 2006):

$$\tau\left(x\right)=\frac{1}{2\sigma\sqrt{\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{1}$$

where μ the best solution is found from the previous iteration and σ is an index of the ant aggregation around the current solution. To initialize the algorithm, μ is chosen lower limited bounds of search space, predefined over continuous decision variables, using a uniform PDF and σ is taken equal to length of the search space to uniformly locate the ants within it. The construction of solutions in this optimization technique (ECACO) on continuous domain is done in principle in the same approach as in the case of standard ACO. The standard deviation of the normal PDF used for the update is modified adaptively based on the index of current iteration and the solutions found in this iteration. To satisfy simultaneously the fitness and aggregation criteria, a concept of weighted standard deviation (Afshar et al. 2006) is defined as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{m} \frac{1}{f_i - f_{opt}} \left[x_i - x_{opt}\right]^2}{\sum_{i=1}^{m} \frac{1}{f_i - f_{opt}}}} \tag{2}$$

where m is the number of ants, x_i is the decision variable created by the ant i (solution), f_i is the fitness value of objective function $f : S \to \Re$ for the solution i, x_{opt} is the best decision variable of the colony in the iteration (best solution) and f_{opt} is the fitness value of the best solution. This approach means that the center of region discovered during the following iteration is the last best point and the narrowness of its width is dependent on the aggregation of the other competitors around the best one. During each iteration the closer the solutions get to the best one, the smaller σ is assigned to the next iteration. It can be noted that since the PDF is normalized within the limited bounds of decision variables, the height of distribution function increase with respect to the previous iteration and its narrowness decreases. Therefore, this strategy concurrently simulates pheromone increase over the promising regions and pheromone evaporation from the others, which are two major characteristics of ant colony pheromone updating rule that described previously (positive and negative update) (Dorigo and Socha 2007).

The error criteria used as the convergence of iterations. Thus, the convergence criterion is defined as $|x_1 - x_0| \leq \varepsilon$ that executed as the termination criterion. ε is a predefined tolerance (specified by the user) for the convergence of iterations. If two consecutive solution values satisfy the proposed criterion, then x_1 is taken as the best solution (x_{opt}) in the corresponding domain. Otherwise, the solution sequence continues with the new starting point in the iterative procedure. In addition, the algorithm stops if the maximum number of iterations (defined by the user) is reached.

Generally, ACO algorithms have been exposed to do better than other heuristic search approaches, including GAs, for small scale problems. Performance of the method, however, deteriorates for problems of growing dimensions (Socha and Dorigo 2008). This can be recognized to the preventing phenomenon which occurs as a result of positive pheromone update rule defined in Eq. 1. In the other word, modification of the PDF characteristics, which can be considered as pheromone information update rule leads to a reduction in the standard deviation of the PDF for the next iteration if the iteration best solution gets better from each iteration to the next iteration. In situations where the iteration best solution of the iteration is worst than that of previous iteration, the standard deviation could increase giving more chance to other regions of the search space to be explored. If the proposed strengthening is very big on some of the options, the ants are very expected to take the next option from among those, because by Eq. 1 the influence of these strengthening considerably biases the probability towards these options. The long-term effect of the proposed problem is sequentially reduced the size of the search space by concentrating on a relatively small number of options. For problems with small search space, the method is more expected to locate the optimal solution before this condition occurs. For large-scale problems, however, the method could encounter oscillatory behavior or premature convergence to sub-optimal solution before optimal solution is located (Afshar et al. 2006; Afshar 2007). In this study, an elitist strategy is observed in the improved ECACO to overcome the proposed problem, in which the best solution of each iteration is directly forwarded to the next generation. This strategy will guarantee that the best solution of the iteration is always improved with respect to the previous iterations and therefore guarantees the convergence of the method and leads to the applicability of that in large-scale and complex optimization problems.

3 Saltwater Intrusion Simulation

3.1 Analytical Solution

The simplified analytical solution of sharp interface saltwater intrusion occurred in the homogenous steady state condition of the unconfined coastal aquifer is used as the tool for simulation in the first approach. The sharp interface approximation is logical in regional scale problems when the transition zone is narrow in relation to the scale of the problem (Reilly and Goodman 1985; Bear et al. 1999). This solution of sharp freshwater–saltwater interface is agreed to the single potential mathematical formulation of Strack (1976). In addition, the Dupuit hydraulic assumption is employed to vertically integrate the flow equation, reducing it from three-dimensional geometry to two-dimensional and the aquifer storativity is unobserved such that the governing equations become time-independent. Similarly, Bear (1979), Naji et al. (1999), Cheng et al. (2000), Mantoglou et al. (2004), Park and Aral (2004), Katsifarakis and Petala (2006) and Mantoglou and Papantoniou (2008) applied proposed approach to simulate the saltwater intrusion.

Figure 2 illustrates a simplified cross section of the sharp freshwater–saltwater interface that normally occurs in the unconfined coastal aquifers (Bear 1979; Naji et al. 1999; Cheng et al. 2000; Mantoglou et al. 2004; Park and Aral 2004; Mantoglou and Papantoniou 2008). It is assumed that the interface has been almost stabilized and is not moving quickly and freshwater flow is horizontal. S denotes the stagnation point representing the region supplying water to the well. Badon–Ghyben–Herzberg principle links h_f (hydraulic freshwater head with reference to the impermeable base of coastal aquifer) to ξ (freshwater depth measured from the sea level) as follows:

$$h_f - d = \delta\xi \tag{3}$$

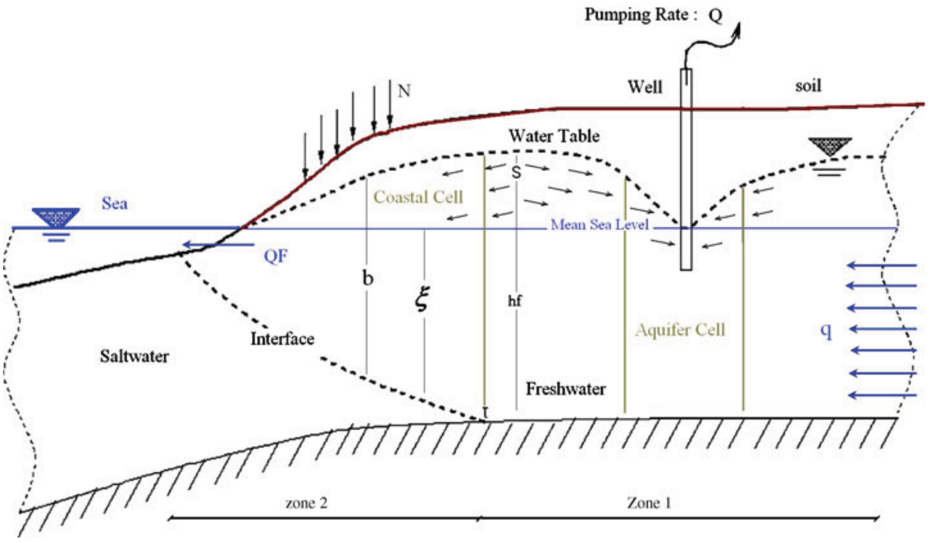


Fig. 2 Sharp freshwater–saltwater interface in the unconfined coastal aquifer

where d is the aquifer depth from its base to mean sea level and δ represents the density difference ratio of the saltwater and freshwater, which is constant throughout this analysis and defined as:

$$\delta = \frac{\rho_s - \rho_f}{\rho_f} \tag{4}$$

where ρ_s and ρ_f are the saltwater and freshwater densities, respectively. The continuity equation of steady flow using Darcy law in the unconfined heterogeneous aquifer is expressed as:

$$\frac{\partial}{\partial x}\left(KB\frac{\partial h_f}{\partial x}\right) + \frac{\partial}{\partial y}\left(KB\frac{\partial h_f}{\partial y}\right) + N - Q = 0 \tag{5}$$

where K is the hydraulic conductivity, N is the surface recharge, Q is total pumping rate, and parameter B defining $B = h_f$ or $B = h_f - d + \xi$ for the zones 1 or 2 of the aquifer, respectively. As outline in Fig. 2, Strack (1976) introduced the flow potential ϕ for unconfined aquifer defined by:

$$\phi = \frac{1}{2}\left[h_f^2 - (1+\delta)\, d^2\right] \text{ zone 1} \tag{6a}$$

$$\phi = \frac{(1+\delta)}{2\delta}\left(h_f - d\right)^2 \text{ zone 2} \tag{6b}$$

For a homogenous aquifer with no pumping or recharge, the flow potential ϕ satisfies the Eq. 5 and simplifies to the Laplace equation $\nabla^2\phi = 0$, in the horizontal x–y plane.

The interface location ξ can be defined using appropriate boundary conditions for unconfined aquifers:

$$\xi = \sqrt{\frac{2\phi}{\delta\,(1+\delta)}} \tag{7}$$

The toe of saltwater can be calculated at $\xi = d$. From Eq. 7, this indicates that the toe is located where ϕ gets the following value:

$$\phi_{toe} = \frac{\delta\,(1+\delta)}{2} d^2 \tag{8}$$

The freshwater potential for multiple pumping wells, in an aquifer with uniform flow, since the Laplace equation is linear, can be obtained using the method of superposition that can be solved either analytically (Strack 1976; Cheng et al. 2000; Park and Aral 2004; Mantoglou and Papantoniou 2008).

$$\phi = \frac{q}{K}x + \sum_{i=1}^{n} \frac{Q_i}{4\pi K} Ln\left[\frac{(x-x_i)^2 + (y-y_i)^2}{(x+x_i)^2 + (y-y_i)^2}\right] \tag{9}$$

where q is the regional uniform freshwater outflow rate per unit length of coastline, and Q_i is the well i discharge located in $(x_i,\ y_i)$ coordinates. Using either Eq. 8 in Eq. 9, the toe location for the multiple wells can be evaluated. In addition, the location of each well stagnation point can be calculated from the Eq. 10 (Bear 1979; Cheng et al. 2000; Park and Aral 2004).

$$\frac{\partial\phi}{\partial x} = 0 \quad and \quad \frac{\partial\phi}{\partial y} = 0 \tag{10}$$

Equation 10 present the nonlinear equations system; illustrate the incredible barrier with no flow velocity (Bear 1979). Newton–Raphson numerical method was used to obtain the stagnation points from proposed equations system that investigated in detail in (Park and Aral 2004). The stagnation point location is operated to distinguish the well saltwater intrusion, by comparing the location of this point corresponding to the toe location on the coastline.

The proposed analytical solution (Eqs. 8–10) constitutes the simulation and constraints within the management model. The simulation equations are treated as required constraints of the optimization model in the present study and produced constraint components within the optimization tool at each iteration of computational procedure.

3.2 Numerical Solution

In this approach, a high degree of complexity is considered. The coastal aquifer simulation model adopted here is a finite difference model for unconfined aquifers with regular or polygonal grids. The planned numerical model incorporates both hydrologic and hydraulic considerations. The aquifer is divided into cells which are selected with regards to the hydrologic properties of the aquifer, of water projects such as wells, hydraulic networks and etc. (Shamir et al. 1984). Assuming that the transition zone is relatively thin and can be considered as a sharp interface,

Fig. 2 illustrates a simplified cross section showing the freshwater–saltwater interface. Coastal cells, alongside the coastline, always contain all of the saltwater intrusion while aquifer cells are regular or polygonal cells and can be defined so as to match with hydraulic systems (wells, water networks, and etc.).

The continuum approach is the standard and as yet most successful way to describe the fundamental processes of physical system in porous media. Therefore, a continuity relation equation in the structure of an implicit finite difference equation, for groundwater flow in each polygonal aquifer cell is (Shamir et al. 1984):

$$\frac{1}{A_i}\sum_j\left[\frac{W_{ji}.T_{ji}}{L_{ji}}\times\left(h_{2j}-h_{2i}\right)\right]+N_i+\frac{R_i+\beta_i\left(DP_i+DG_i\right)-PP_i-PG_i}{A_i}$$
$$+\frac{q\times W_i}{A_i}=S_i\times\left(\frac{h_{2i}-h_{1i}}{\Delta t}\right) \tag{11}$$

where A_i is cell i area, W_{ji} and T_{ji} representing the length of the boundary and transmissivity between cell i and an adjacent cell j, L_{ji} is the distance between centers of the two adjacent cells, N_i is natural recharge, R_i is artificial recharge, S_i is storativity in the cell i, Δt is length of time period, DP_i and DG_i are supply to private and government consumers, respectively, PP_i and PG_i are private and government pumping, respectively, β_i is the fraction of supply which reaches the groundwater as return flow, h_{1i} and h_{2i} are groundwater level at the beginning and end, respectively, of the time period and q is the freshwater flow per unit width into boundary aquifer cells.

Similarly, continuity equation for the coastal cells can be formulated as (Shamir et al. 1984):

$$\frac{1}{A_i}\sum_j\left[\frac{W_{ji}.T_{ji}}{L_{ji}}\times\left(h_{2j}-h_{2i}\right)\right]+N_i-\frac{W_i.QF_i}{A_i}=S_i\times\left(\frac{h_{2i}-h_{1i}}{\Delta t}\right) \tag{12}$$

where W_i is the width of coastal cell along the coast and QF_i is the freshwater flow per unit width into the sea at the coastline from coastal cell i. QF_i is predictable by Eq. 13 (Bear 1979):

$$QF_i=\frac{2\times T_i\times\left(h_{1i}-d\right)}{L_i} \tag{13}$$

where d is the mean sea level above the datum, T_i and L_i are mean transmissivity and mean saltwater intrusion length in the coastal cell i. A continuity equation, formulated for the hydraulic system in each aquifer cell (Shamir et al. 1984) is given by:

$$PP_i+PM_i-R_i+Q_i^{trnsf}=DP_i+DM_i \tag{14}$$

where Q_i^{trnsf} is net import flow rate to hydraulic system presented in the cell i.

A segment of the coastal aquifer formulation presented here, is the general procedure and models the real and complex coastal aquifer. Validity of this approach is checked with the help of an analytical solution, as described the previous section.

Therefore, in this study, steady state condition with some simplification is assumed. In the proposed condition, Eqs. 11 and 12 as follows, respectively:

$$N_i + \frac{R_i + \beta_i\left(DP_i + DM_i\right) - PP_i - PM_i}{A_i} + \frac{q \times W_i}{A_i} = \frac{1}{A_i}\sum_j \left[\frac{W_{ji}.T_{ji}}{L_{ji}} \times \left(h_i - h_j\right)\right] \tag{15a}$$

$$N_i + \frac{2 \times W_i \times T_i \times d}{A_i \times L_i} = \frac{1}{A_i}\sum_j \left[\frac{W_{ji}.T_{ji}}{L_{ji}} \times \left(h_i - h_j\right)\right] + \frac{2 \times W_i \times T_i \times h_i}{A_i \times L_i} \tag{15b}$$

For the implicit option, Eqs. 15a and 15b are solved simultaneously through an iterative strategy; the proposed equations possibly summarized in the matrix form of $[B] = [A].[h]_i$. The proposed model uses double precision throughout in order that round-off error is assumed to be negligibly small.

4 Management Problem Formulations

The objective of the optimization model is to maximize total water-pumping rate from the existing wells in the coastal aquifer while protecting from saltwater intrusion in addition to pumping considerations. Groundwater pumping rates are defined as decision variables and exploit from pre-selected pumping wells, that the coordinates of those (x_i, y_i); $i = 1, \ldots, n$ are assumed known. Where n is the total number of pumping wells. Mathematically, the saltwater intrusion management problem may be formulated as follows:

$$Maximize \quad F : Q_{total} = \sum_{i=1}^{n} Q_i \tag{16}$$

Subject to:

$$\begin{aligned} Q_i^{\min} \le Q_i \le Q_i^{\max} \quad & i = 1, \ldots, n \\ x_i^{toe} < x_i^{S} \qquad & i = 1, \ldots, n \end{aligned} \tag{17}$$

where $Q_i^{\min}$ and $Q_i^{\max}$are the minimum and maximum allowable pumped water of well i, respectively. Pumping limits from the wells are associated with water demand, equipment facilities, exploitation restriction, and other operational considerations. x_i^{toe} and x_i^{S} are the distance from the coastline to the toe and stagnation points of the pumping well i, respectively. These constraints keep the wells located near the coast from saltwater intrusion threat by not permitting the toe of the interface to contact the stagnation points of the wells. The pumping information considered in the optimization model is applied to calculate x_i^{toe} and x_i^{s} distances in the simulation tool, that implicitly depend on the pumping rates, nonlinearly. Therefore, the objective function (Eq. 16) along with the constraints (Eq. 17) constitute a nonlinear optimization problem. The simulation model is a key role in this part due to find feasible solutions.

5 Solution Approach and Methodology

The analytical and numerical solution of the saltwater intrusion and ECACO are combined to find the optimal solution of coastal aquifer management problem. Most search methods, including EACO, find feasible solutions of the constrained problem by penalizing infeasibilities to force the search towards the feasible region. The underlying constrained problem is transformed to an unconstrained one, using the penalty function and building a single objective function, which in turn is optimized using an unconstrained optimization algorithm (Dorigo and Stützle 2004; Afshar 2007).

To allow an ECACO to be used, the inequality constrained problem in Eqs. 16 and 17 is first transformed into an unconstrained one. This is accomplished by introducing the penalty function as a decision variable into the ECACO search. The value of the penalty function is regulated heuristically during the evolutionary process. To ensure no saltwater intrusion, a normalized modified objective function is assigned to the proposed problem as:

$$Maximize \quad F = \sum_{i=1}^{n} \left[\frac{Q_i}{Q_i^{\max}} \right] - C_{Sc} \times Pen \tag{18}$$

where C_{Sc} is a scale coefficient and is zero when the constraint $x_i^{toe} < x_i^S$ is satisfied and gets a proper value when this constraint is violated. *Pen* is the penalty function as follows:

$$Pen = \sum_{i=1}^{n} \left[\frac{x_i^{toe}}{x_i^S} - 1 \right] \tag{19}$$

The constraint $Q_i^{\min} \leq Q_i \leq Q_i^{\max}$ is not included in *Pen* as it is automatically satisfied in an ECACO by properly selecting the search space of Q_i and forcing the

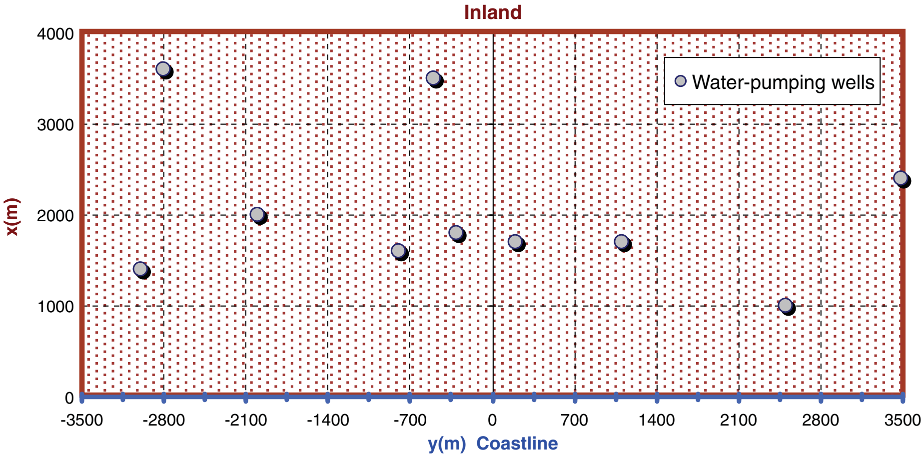


Fig. 3 The study area used in the simulation–optimization procedure

Table 2 Summary of the coastal aquifer system properties

Aquifer property	Value[a]
Aquifer type	Unconfined
Saltwater density (kg/m^3)	1.025
Freshwater density (kg/m^3)	1
Saltwater depth (m)	15
Hydraulic conductivity (m/day)	40
Uniform flow rate in the negative x-axis direction (m^3/day/m)	0.4015

[a] The values adopted from Cheng et al. (2000) and Park and Aral (2004)

optimization algorithm to identify optimal solution in this space. F is then maximized in an unconstrained optimization procedure. It should be mentioned that the results of the evaluation procedure depend heavily on the form of the penalty function. It should be properly selected, in order to ensure observance of the constraints, without obscuring the optimization target. Optimal solution of algorithm is feasible and satisfactory, when the penalty function value in the final iteration is zero. This condition should be checked for all combined algorithm applications.

It is worth mentioning that under steady state flow conditions, by increasing the water pumping, saltwater intrudes the aquifer through coastline sections with a net water inflow, and eventually enters into the wells. If no such conditions exist and no contact between the toe of saltwater interface and the stagnation points of the wells, there is no saltwater intrusion to the wells (Katsifarakis and Petala 2006). It is satisfactory then for proposed condition, consistent and meaningful approximations and simplifications in the simulation procedure that discussed previously. The pumping information from the optimization model is passed to the simulation part of the combined algorithm. The simulation model simulates the stagnation points of the wells and toe location of the saltwater interface, and subsequently this information are returned to the optimization model. This information is used to calculate Pen using Eq. 19.

In the numerical simulation approach, the study area has been divided into aquifer and coastal cells (grids) and pressure of wells or other hydraulic systems to the groundwater system in the coastal aquifer are averaged in each aquifer cell and the constraint ($x_i^{toe} < x_i^S$) satisfied by dictating to the model with set up the desired saltwater intrusion toe locations as the coastal cells length.

Table 3 Optimal solution (pumping rate policy) of the management problem for example 1

Well number	x_w(m)	y_w(m)	Optimal pumping rate (m^3/day)		
			This work (analytical)	Cheng et al. (2000)	Park and Aral (2004)
1	1,000	2,500	197.6	201	198.1
2	1,700	1,100	386.0	350	380
3	1,700	200	150.1	150	150.1
4	3,500	−500	1,460.6	1,497	1,462
5	2,000	−2,000	150.2	155	150.0
6	3,600	−2,800	1,406.9	1,387	1,406.6
7	1,400	−3,000	150.1	150	150.2
Total			3,901.5	3,890	3,897.0

Table 4 Optimal solution (pumping rate policy) of the management problem for example 2

Well number	x_w (m)	y_w (m)	Optimal pumping rate (m^3/day)		
			This work (analytical)	Cheng et al. (2000)	Park and Aral (2004)
1	1,000	2,500	222.8	255	221.7
2	1,700	1,100	587.6	402	579.8
3	1,800	−300	150.2	158	154.4
4	3,500	−500	658.5	728	733.2
5	1,600	−800	150.1	150	151.1
6	3,600	−2,800	1,499.9	1,500	1,402.9
7	1,400	−3,000	197.0	185	215.9
8	2,000	−2,000	203.1	232	178.4
Total			3,669.2	3,610	3,637.4

6 Model Application and Performance Evaluations

The applicability and efficiency of the developed analytical formulations described in the previous sections based on ECACO are tested in this section against a number of examples in the literature and the developed model is applied to the optimal management of a 4,000 m × 7,000 m coastal aquifer. A schematic of the test problem is shown in Fig. 3. A number of wells pump water from a homogeneous and isotropic unconfined aquifer of rectangular shape that is affected by saltwater intrusion along the coastal side of the study area. The properties of the coastal aquifer system are given in Table 2. The application of the proposed optimization model allows finding the optimal pumping pattern in order to satisfy the water demand needs

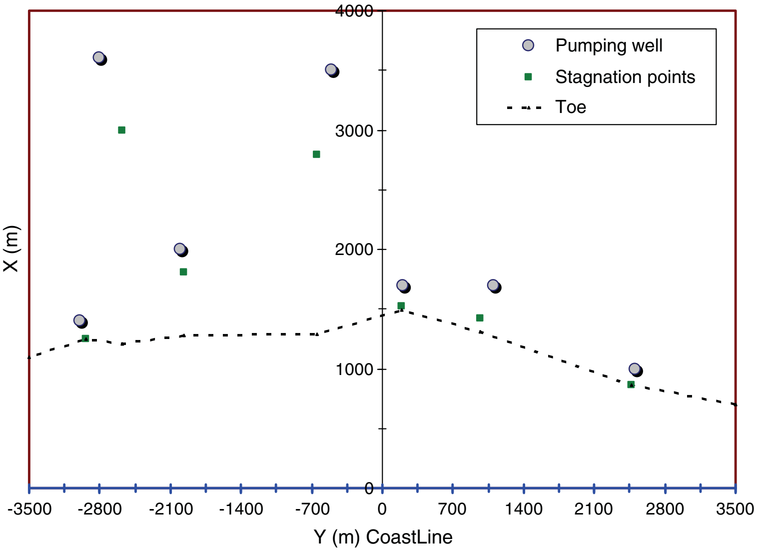


Fig. 4 The toe location and the stagnation point of the wells for example 1

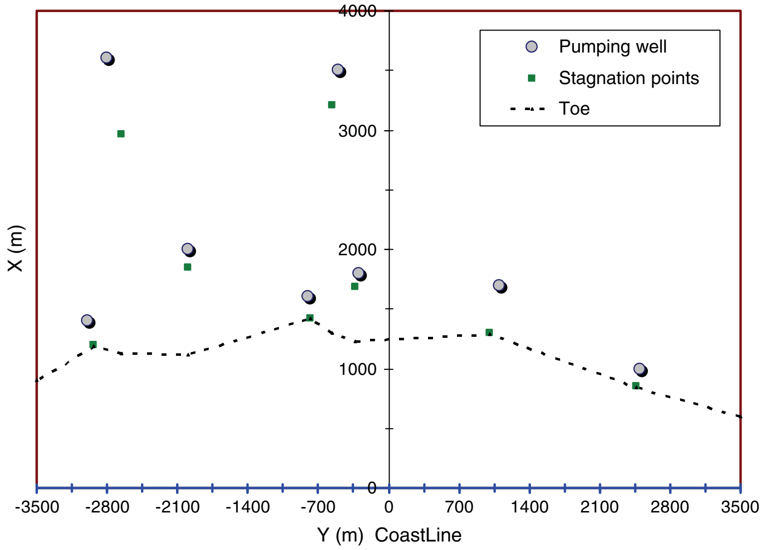


Fig. 5 The toe location and the stagnation point of the wells for example 2

and to control the saltwater intrusion in the aquifer. Finally, the obtained results demonstrate model capability to solve practical problems.

To verify the present developed ECACO-based model, three examples that have been studied by Cheng et al. (2000) and Park and Aral (2004), are adopted in this

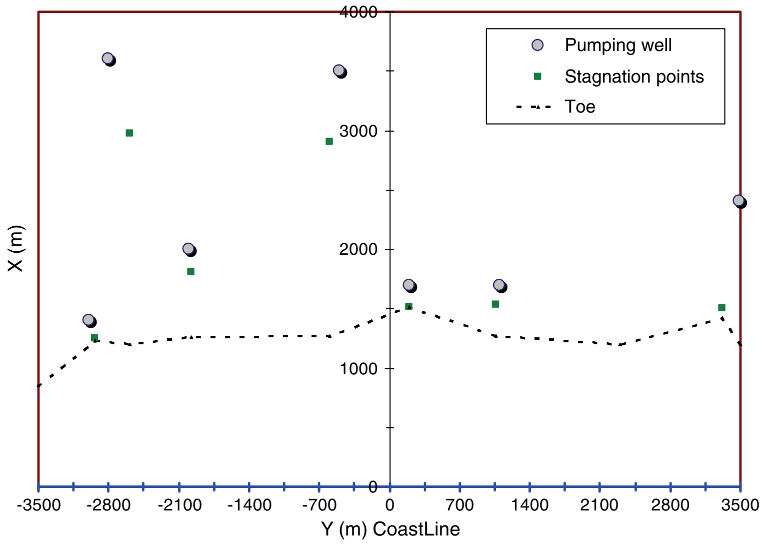


Fig. 6 The toe location and the stagnation point of the wells for example 3

Table 5 Optimal solution (pumping rate policy) of the management problem for example 3

Well number	x_w (m)	y_w (m)	Optimal pumping rate (m^3/day)	
			This work (analytical)	Park and Aral (2004)
1	2,403.4	3,492	1,499.2	1,477.4
2	1,700	1,100	196.6	166.4
3	1,700	200	159.5	150.1
4	3,500	−500	1,043.8	1,220.7
5	2,000	−2,000	150.2	186.9
6	3,600	−2,800	1,495.6	1,275.1
7	1,400	−3,000	150.2	166.2
Total			4,695.0	4,642.4

paper. For each well, lower and upper limited bound of pumping well ($Q^{\min}$ and $Q^{\max}$) are assumed to be 150 m^3/day and 1,500 m^3/day, respectively. The pumping rates of all wells as the starting points in the optimal approach initially set to the minimum pumping rate (150 m^3/day). The value of parameter δ is taken as 0.025. The flow potential at the toe location of saltwater is computed in accordance with Eq. 8 equal to 2.883 m^2. Also, the stagnation points of the existing wells and toe locations are calculated.

The example 1 consists of seven wells and the example 2 contains eight wells. Tables 3 and 4 compare the optimal solution (pumping operation policy) obtained by the proposed combined algorithm with those presented by Cheng et al. (2000) and Park and Aral (2004). In example 1, the total pumping rate is obtained 3,901.5 m^3/day while this value is reported 3,890 and 3,897 m^3/day in by Cheng et al. (2000) and Park and Aral (2004), respectively. Furthermore, in example 2, the total pumping rate is reached to 3,669.2 m^3/day. Cheng et al. (2000) and Park and Aral (2004) give 3,610 and 3,637 m^3/day for this value. It can be seen ECACO can be applied successfully for the problem of pumping optimization in saltwater-intruded coastal aquifers. Also, the results show a small amount improvement in the total pumping rates and perhaps the superiorities of the ECACO approach, comparatively. The toe location and the stagnation point of the wells in the coastal aquifer are illustrated in Figs. 4 and 5 for

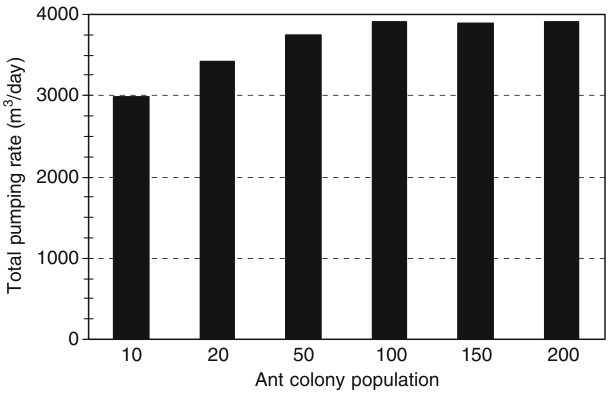
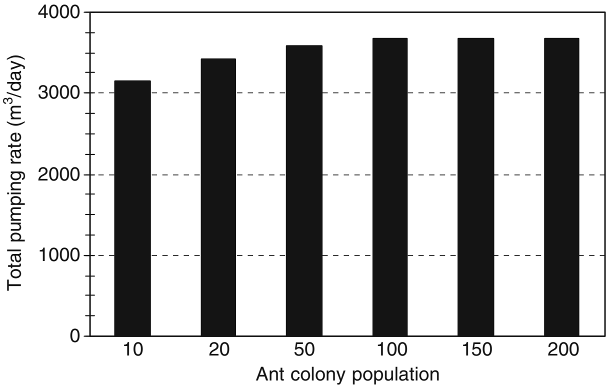


Fig. 7 Total pumping rate of example 1 for different ant populations

Fig. 8 Total pumping rate of example 2 for different ant populations



example 1 and 2, respectively. Example 3 is chosen the result of the optimization of well location cases from Park and Aral (2004) work and in the same way; optimal pumping rate is obtained for this example. Hence, total pumping rate is calculated 4,695.0 m^3/day while Park and Aral (2004) obtained 4,642.4 m^3/day for this case. Figure 6 and Table 5 summarize the result of this optimal approach.

ECACO algorithm by considering a normalized modified objective function Eq. 18 is used for finding optimal solution of constrained management problem. Here improved ECACO algorithm in combination with simulation approach, are evaluated for different ant colony population of 10, 20, 50, 100, 150, and 200 to assess the effect of the ant population on the performance of the algorithm. All these trials are carried out using the 200 iterations obtained after some preface runs. Figures 7 and 8 evaluate the value of total pumping rate for example 1 and 2, based on 10 runs obtained with the improved ECACO, in the simulation–optimization approach for different ant populations.

It is seen from these Figs. that the quality of optimal solutions improved with increasing the ant populations whereas reached the optimum required ants. The comparisons and reported results are based on 100 ant colony populations. The mean, minimum, and maximum values of the objective function with the proposed

Table 6 Mean, maximum, and minimum of optimal solution (pumping rate policy) for example 1

Well number	x_w(m)	y_w(m)	Optimal pumping rate[a] (m^3/day)		
			Mean	Min	Max
1	1,000	2,500	197.6	201.8	199.3
2	1,700	1,100	386.0	355.1	385.1
3	1,700	200	150.1	156.1	150.0
4	3,500	−500	1,460.6	1,495.9	1,461.5
5	2,000	−2,000	150.2	160.8	150.0
6	3,600	−2,800	1,406.9	1,373.8	1,407.2
7	1,400	−3,000	150.1	150.2	150.0
Total			3,901.5	3,893.7	3,903.1

[a] Based on 10 independent runs and 100 ant colony populations

Table 7 Mean, maximum, and minimum of optimal solution (pumping rate policy) for example 2

Well number	x_w(m)	y_w(m)	Optimal pumping rate[a] (m^3/day)		
			Mean	Min	Max
1	1,000	2,500	222.8	219.9	220.5
2	1,700	1,100	587.6	562.7	576.9
3	1,800	−300	150.2	150.0	150.1
4	3,500	−500	658.5	868.6	723.7
5	1,600	−800	150.1	150.3	150.1
6	3,600	−2,800	1,499.9	1,289.2	1,499.5
7	1,400	−3,000	197.0	242.6	207.3
8	2,000	−2,000	203.1	150.9	150.4
Total			3,669.2	3,634.3	3,678.6

[a] Based on 10 independent runs and 100 ant colony populations

number of ants obtained are listed in Tables 6 and 7. As regards proposed results, convergence characteristics of the applied ECACO algorithm are exposed.

Plots of dimensionless objective function value vs. 50 first iteration for example 1 and 2 are shown in Figs. 9 and 10, respectively. Objective values in early iterations have the fast improvement, because the penalty term in objective function Eq. 18 is zero, due to set the initial values of decision variables equal to lower limited bounds of search space. This is followed by much slower improvement in later iterations or same objective values in the several iterations due to implement elitist strategy in the ECACO algorithm, as better feasible solutions are found.

Optimal solutions were obtained by Cheng et al. (2000) on a Pentium 450 MHz microcomputer using about 6 h of CPU time, while Park and Aral (2004) used less than 30 min of simulation–optimization time on a compatible machine to obtain optimal results. The time required for each run in the present simulation–optimization approach and obtaining optimal solution, is approximately 2 min using nominal 2.03 GHz AMD Athlon (tm) XP processor and 512 MB RAM, less than both previous works.

Additionally, in this section, the performance of the described numerical simulation methodology in combination with ECACO algorithm is evaluated. The main objective of these performance evaluations is to establish the feasibility of the

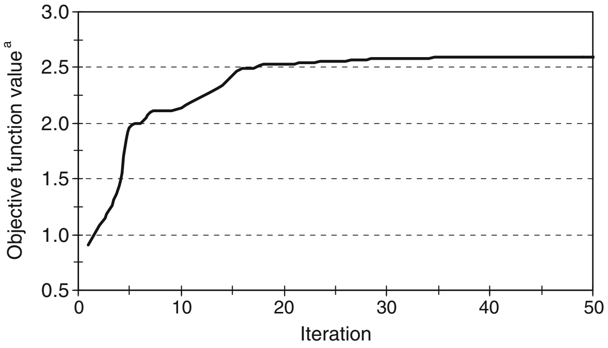


Fig. 9 Convergence of normalized objective function using 100 population ants for example 1. ^{a} denotes the normalized objective function referred to Eq. 18

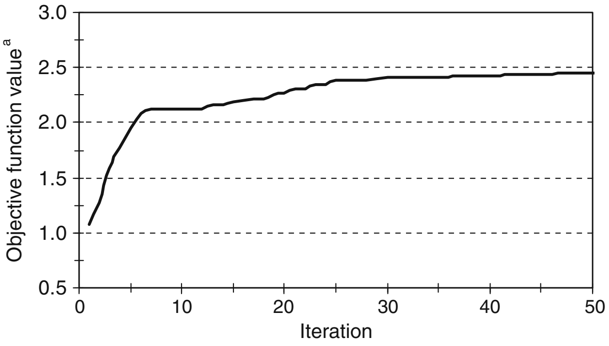


Fig. 10 Convergence of normalized objective function using 100 population ants for example 2. ^{a} denotes the normalized objective function referred to Eq. 18

numerical simulation–optimization methodology. In addition, the work presented here is of exploratory nature for developing new techniques, the proposed management model will be tested on a hypothetical aquifer relatively small in size. Hence, an aquifer having a simple rectangular geometry is selected in order to test and compare the proposed second management approach. All aquifer physical parameters selected are similar to the one given in the application examples considered in the first approach except for the well locations. Well locations are corrected by a few changes due to discretization in the numerical scheme. The study area of interest is 28 km^2 as shown in Fig. 11 and is covered with a two-dimensional grid. The sizes of each cell are $\Delta x = 1{,}000$ m and $\Delta y = 1{,}000$ m. Because of simulating infinite study area in the three inland faces, the geometric dimensions of the aquifer are assumed 1,000 m × 5,000 m. The bottom face represents the sea face. Opposite to the sea face is the inland face, which is a source of freshwater. The right and left faces of the aquifer are assumed to be unlimited through increasing the 2,000 and 1,000 m of study area dimensions in the left and right face, respectively. Decision variables of the management model are the pumping rates subject to specified lower and upper bounds. Note that the same constraint and ECACO parameters set defined in the first approach are also valid

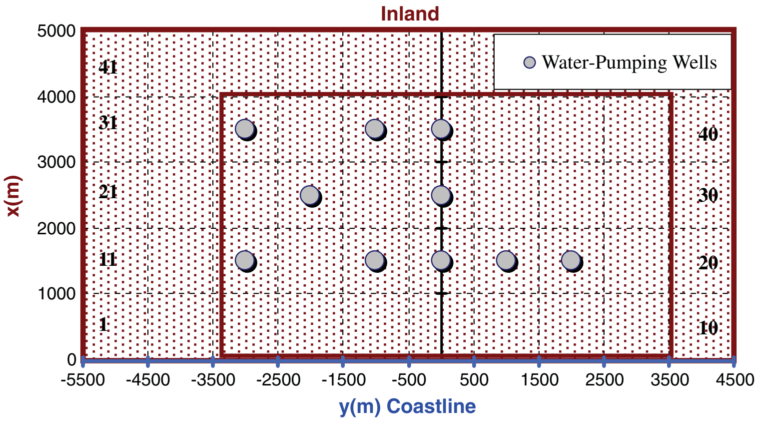


Fig. 11 Schematic view of cells and the boundaries of the coastal area

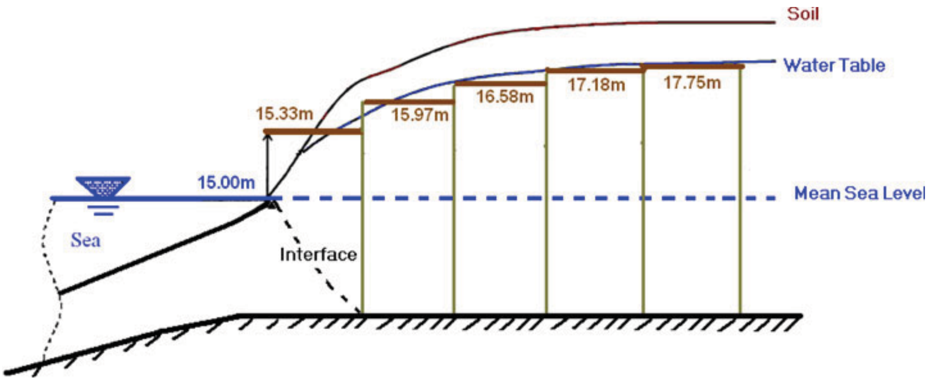


Fig. 12 Groundwater heads in the no pumping steady state condition (calculated in the numerical approach)

for this numerical based approach. Considering the properties of the coastal aquifer system, groundwater head in the no-pumping condition can be calculated using numerical solution, as described earlier. Figure 12 presents the groundwater heads in the no-pumping steady state condition. The desired saltwater location distance is assumed to be 1,000 m (coastal aquifer length). Extra parameters that observed in the numerical simulation are ignoring in this examples. The example 4 consists of seven wells and the example 5 contains eight wells, comparable with examples 1 and 2, respectively. The results of the first method application for the examples 4 and 5 are demonstrated in Tables 8 and 9. Also, Tables 8 and 9 compare the results of the first model with the results of the current approach. As can be seen from Table 8, the numerical-based approach found a total mumping rate value of 3,696.31 m^3/day and this result closely agrees with the first analytical solution (3,695.1 m^3/day). In example 5, the total pumping rate is obtained 3,681.08 m^3/day while this value is calculated 3,624.33 m^3/day by the first approach. The numerical results obtained for the considered examples compatible with the first work. The key difference between the first and the second management approaches is that the second management model can be considered more assumptions that truly simulate real coastal aquifers.

Table 8 Optimal solution (pumping rate policy) of the management problem for example 4

Well number	x_w(m)	y_w(m)	Optimal pumping rate (m^3/day)	
			Analytical approach	Numerical approach
1	3,500	−3,000	1,499.40	1,303.35
2	3,500	−1,000	977.75	959.98
3	2,500	−2,000	150.00	150.08
4	1,500	−3,000	150.00	150.15
5	1,500	0	163.10	193.37
6	1,500	1,000	182.32	231.04
7	1,500	2,000	572.53	708.34
Total			3,695.1	3,696.31

Table 9 Optimal solution (pumping rate policy) of the management problem for example 5

Well number	x_w(m)	y_w(m)	Optimal pumping rate (m^3/day)	
			Analytical approach	Numerical approach
1	3,500	−3,000	1,500.00	1,274.34
2	3,500	0	704.53	899.57
3	2,500	−2,000	150.00	157.64
4	2,500	0	150.00	160.33
5	1,500	−3,000	236.59	215.08
6	1,500	−1,000	150.00	150.02
7	1,500	1,000	178.00	191.01
8	1,500	2,000	555.21	633.10
Total			3,624.33	3,681.08

7 Conclusion

The extreme pumping of groundwater in many coastal aquifers causes the intrusion of saltwater and the deterioration of water quality. Water management models are necessary for design sustainable and proper groundwater operation strategies to meet demands, while controlling the saltwater intrusion process through planned pumping. A new approach based on Elitist Continuous Ant Colony Optimization (ECACO) algorithm in cooperation with simplified analytical and the numerical simulation of saltwater intrusion problem is proposed in this paper. Proposed ECACO algorithm uses a modified objective function based on heuristic pheromone assignment approach for pheromone update to filter optimal solution candidates. Optimal solutions can be reached more rapidly by self-adjusting the path searching behavior of the ants according to objective values. In the adopted approach, consistent penalty function is incorporated in formulation of the coastal aquifer management, and three experiments are performed on the previous works of Cheng et al. (2000) and Park and Aral (2004). Furthermore, the applicability and efficiency of procedure is investigated. The obtained results are reported and compared with those obtained through proposed works. The results indicate the ability of ECACO algorithm in comparison with proposed studied, regarding convergence characteristics and success rate, time requirement for computation and the final solution quality. Additionally, two application examples with equality and inequality constraints are adopted from the described experiments for numerical simulation approach. In order to model the aquifer system on numerical model, the study area is discretized into regular rectangular block centered finite difference grid blocks. It can be noted that polygonal discretization is available in the proposed model. For comparison purposes and to demonstrate the differences in the solutions, ECACO, uses penalty function approach is also applied to both examples. This application transforms the model into an unconstrained problem and the constraints are satisfied to some degree through the introduction of a penalty function. The comparison of the second approach results is reasonable with those of the first approach.

It can be concluded the evolutionary search methods, in particular GAs or ECACOs, have been successfully used for non-linear complex optimization problems. Therefore, the ECACO appears to be a useful technique for solving constrained optimization problems in coastal aquifer management problems, derived from the results obtained in this study. A preliminary application of numerical simulation-

ECACO algorithm to hypothetical cases indicates that the model can be a useful tool for optimal management of groundwater in coastal areas. The numerical model is more general than previous analytical solutions (Cheng et al. 2000; Mantoglou 2003; Mantoglou et al. 2004; Park and Aral 2004) and it can handle aquifers of complex conditions to prevent saltwater intrusion. To conclude, a further possibility to explore is that of integrating the proposed methodology approach, based on the formalization of optimal decision problems, with the developed numerical approach in the present study or commercially available simulation tools.

Notation

The following symbols are used in this paper:

μ	the best solution is found from the previous iteration of optimization procedure
σ	an index of the ant aggregation around the solution of the current iteration
τ	normal distribution for calculating pheromone information
x_i	the decision variable created by the ant (solution)
f_i	the fitness value of objective function
$f : S \rightarrow \Re$	objective function
x_{opt}	the best decision variable of the colony in the iteration (best solution)
f_{opt}	the fitness value of the best solution
h_f	hydraulic freshwater head with reference to the impermeable base of coastal aquifer
ξ	freshwater depth measured from the sea level
d	the aquifer depth from its base to mean sea level
δ	the density difference ratio of the saltwater and freshwater
ρ_s	the saltwater density
ρ_f	the freshwater density
K	the hydraulic conductivity
N	the surface recharge (natural)
Q	total pumping rate
ϕ	the flow potential
q	the regional uniform freshwater outflow rate per unit length of coastline
A	the cell area
W_{ji}	the length of the boundary between cell i and an adjacent cell j
T_{ji}	the transmissivity between cell i and an adjacent cell j
L_{ji}	the distance between centers of the two adjacent cells
R	artificial recharge
S	storativity
Δt	length of time period
DP	supply to private consumers
DG	supply to government consumers
PP	private pumping
PG	government pumping
β	the fraction of supply which reaches the groundwater as return flow
h_{1i}	groundwater level at the beginning of the time period

h_{2i}	groundwater level at the end of the time period
W	the width of coastal cell along the coast
QF	the freshwater flow per unit width into the sea at the coastline from coastal cell
Q^{trnsf}	net import flow rate to hydraulic system presented in the cell
x^{toe}	the distance from the coastline to the saltwater toe
x^{S}	the distance from the coastline to the stagnation points of the pumping well
Pen	the penalty function

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