CHAPTER 1

Basic Principles

1.1 INTRODUCTION

Open channel hydraulics is the study of the physics of fluid flow in conveyances in which the flowing fluid forms a free surface and is driven by gravity. The primary case of interest in this book is water as the flowing fluid having an interface or free surface formed with the ambient atmosphere, but the basic principles also apply to other cases such as density-stratified flows. Natural open channels include brooks, streams, rivers, and estuaries. Artificial open channels are exemplified by storm sewers, sanitary sewers, and culverts flowing partly full, as well as drainage ditches, irrigation canals, aqueducts, and flood diversion channels. Applications of open channel hydraulics range from the design of artificial channels for beneficial purposes such as irrigation, drainage, water supply, and wastewater conveyance to the analysis of flooding in natural waterways to delineate floodplains and assess flood damages for a flood of specified frequency. Principles of open channel hydraulics also are utilized to describe the transport and fate of environmental contaminants, including those carried by sediments in motion, as well as to predict flood surges caused by dam breaks or hurricanes.

1.2 CHARACTERISTICS OF OPEN CHANNEL FLOW

Although the basic principles of fluid mechanics are applicable to open channel flow, such flow is considerably more complex than closed conduit flow due to the free surface. The relevant forces causing and resisting motion and the inertia must form a balance such that the free surface is a streamline along which the pressure is constant and equal to atmospheric pressure. This extra degree of freedom in open channel flow means that the flow boundaries no longer are fixed by the conduit geometry, as in closed conduit flow, but rather the free surface adjusts itself to accommodate the given flow conditions.

Another important characteristic of open channel flow is the extreme variability encountered in cross-sectional shape and roughness. Conditions range from a circular gravity sewer flowing partly full to a natural river channel with a floodplain subject to overbank flow. Roughness heights in the gravity sewer correspond to those encountered in closed conduit flow, while roughness elements such as brush, vegetation, and deadfalls in natural open channels make the roughness extremely difficult to quantify. Even in the case of the circular gravity sewer, resistance to flow is complicated by the change in cross-sectional shape as the depth changes. In alluvial channels, the boundary itself is movable, giving rise to bed forms that provide a further contribution to flow resistance.

Because of the free surface, gravity is the driving force in open channel flow. The ratio of inertial to gravity forces in open channel flow is the most important governing dimensionless parameter. It is called the *Froude number,* defined by

$$\mathbf{F} = \frac{V}{(gD)^{1/2}} \tag{1.1}$$

in which V is the mean velocity, D is a length scale related to depth, and g is gravitational acceleration. In some instances the Reynolds number also is important, as in closed conduit flow, but one of the few simplifications in natural open channels is the existence of a large Reynolds number so that viscous effects assume less importance. Flow resistance in this case can be dominated by form resistance, which is associated with asymmetric pressure distributions resulting from flow separation. The success of Manning's equation in characterizing open channel flow resistance in fact depends on the existence of a Reynolds number large enough that the Manning's resistance factor is invariant with Reynolds number.

1.3 SOLUTION OF OPEN CHANNEL FLOW PROBLEMS

The complexities offered by open channel flow often can be dealt with through a combination of theory and experiment, as in other branches of fluid mechanics. The basic principles of continuity, energy conservation, and force-momentum flux balance must be satisfied, but we often must resort to experiments to complete the solution of the problem. The resulting relationships can be quite complicated, especially when the variability of the cross-sectional geometry is considered.

In the not-too-distant past, the design of open channels was achieved with the aid of numerous nomographs and graphical relationships because of the nonlinearity of the governing equations combined with complex geometry. More extensive analysis of unsteady flow problems or gradually varied flow problems associated with river floodplains required mainframe computers. Presently, the proliferation of personal computers and engineering workstations has provided much greater accessibility and flexibility for simple as well as complex problems in open channel hydraulics. Programs that are truly interactive with immediate feedback of results in the form of screen graphics can be written with ease. The hydraulic engineer can investigate a wide array of design solutions and their implications in a completely interactive mode in the modern engineering workstation. On the other hand, such ease of use sometimes leads to misinformed applications of accepted programs that have been transported from the mainframe to personal computers.

1.4 PURPOSE

The theme of this book is to present modern numerical techniques for the solution of open channel flow problems in the current personal computing environment as well as to emphasize experimental results and their application in free surface flows. The problem of a variable bed surface caused by sediment transport in alluvial channels is treated as well. In addition, focus is placed on the application of basic principles of fluid mechanics to the formulation of open channel flow problems, so that the assumptions and limitations of the numerical models now widely available are made clear. The combination of theoretical, experimental, and numerical techniques applied to open channel flow provides a synthesis that has become the hallmark of modern fluid mechanics.

1.5 HISTORICAL BACKGROUND

The following discussion relies on the excellent historical treatment of hydraulics by Rouse and Ince (1957), to which the reader is referred for further details.

From the advent of civilization, the conveyance of water in open channels has been used to meet basic needs, such as irrigation for the Egyptians and Mesopotamians, water supply for the Romans, and waste disposal for Europeans in the Middle Ages, with the disastrous results of waterborne disease transmission. In some cases, artificial open channels were constructed, while in others natural river channels were utilized to convey water and wastes.

The Egyptians used a dam for water diversion and gravity flow through canals to distribute water from the Nile River, and the Mesopotamians developed canals to transfer water from the Euphrates to the Tigris rivers, but there is no recorded evidence of any understanding of the theoretical flow principles involved. The Chinese are known to have devised a system of dikes for protection from flooding several thousand years ago. Evidence of water supply pipes and brick conduits for drainage dated to 3000 years B.C. has been found in the Indus River valley. The success of these early, extensive hydraulic works was likely the result of experience only.

Roman aqueducts were used to transport water from springs to distribution reservoirs. The aqueducts were rectangular, masonry canals supported by masonry arches, and they conformed to the natural topography in longitudinal slope. The water discharge in the aqueducts was measured as the cross-sectional area of flow with no regard for the velocity or slope producing the velocity. Although the existence of a conservation principle was recognized, the conserved quantity of volume flux was misunderstood. Yet, these aqueducts served their engineering purpose, albeit inefficiently and uneconomically in modern terms.

The philosophical approach of the Greeks toward physical phenomena was revived by the Scholasticism of the Middle Ages, and it remained for Leonardo da Vinci to introduce the experimental method in open channel flow during the Renaissance. Leonardo's prolific writings included observations of the velocity distribution in rivers and a correct understanding of the continuity principle in streams with narrowing width. Some early experimental results on pipe and channel flow were reported by Du Buat in 1816, but the experimental work on canals begun by Darcy and completed by Bazin in the late 19th century, and Bazin's experiments on weirs, were unsurpassed at the time and remain an enduring legacy to the experimental approach.

The problem of open channel flow resistance was recognized as important by many engineers in the 18th and 19th centuries. The work of Chezy on flow resistance began in 1768, originating from an engineering problem of sizing a canal to deliver water from the Yvette River to Paris. The resistance coefficient attributed to him, however, was introduced much later because his work dealt only with ratios of the independent variables of slope and hydraulic radius to the 1/2 power in a relationship for velocity ratios in different streams. His work was not published until the 19th century. The Manning equation for open channel flow resistance, about which much will be said in this book, has a complex historical development but was based on field observations. The Irish engineer Robert Manning actually discarded the formula because of its nonhomogeneity in favor of a more complex one in 1889, and Gauckler in 1868 preceded Manning in introducing a formula of the type that now bears the name of Manning.

The theoretical approach to open channel flow rests on the firm foundation built by Newton, Leibniz, Bernoulli, and Euler, as in other branches of fluid mechanics; but one of its early fruits was the analytical solution of the equation of gradually varied flow by Bresse in 1860 and the correct formulation of the momentum equation for the hydraulic jump, which he attributed to the 1838 lecture notes of Belanger. In addition, Julius Weisbach extended the sharp-crested weir equation in 1841 to a form similar to that used today. By the end of the 19th century, many of the elements of the modern approach to open channel flow, which includes both theory and experiment, had been established.

The work of Bakhmeteff, a Russian emigre to the United States, had perhaps the most important influence on the development of open channel hydraulics in the early 20th century. Of course, the foundations of modern fluid mechanics (boundary layer theory, turbulent velocity and resistance laws) were being laid by Prandtl and his students, including Blasius and von Kármán, but Bakhmeteff's contributions dealt specifically with open channel flow. In 1932, his book on the subject was published, based on his earlier 1912 notes developed in Russia (Bakhmeteff 1932). His book concentrated on "varied flow" and introduced the notion of specific energy, still an important tool for the analysis of open channel flow problems. In Germany at this time, the contributions of Rehbock to weir flow also were proceeding, providing the basis for many further weir experiments and weir formulas.

By the mid-20th century, many of the gains in knowledge in open channel flow had been consolidated and extended in the books by Rouse (1950), Chow (1959), and Henderson (1966), in which extensive references can be found. These books set the stage for applications of modern numerical analysis techniques and experimental instrumentation to problems of open channel flow.

1.6 DEFINITIONS

In a steady open channel flow, the depth and velocity at a point do not change as a function of time. In the more general case of unsteady flow, both velocity and depth vary with time, as in the case of the passage of a flood wave in a river as shown in Figure 1.1a relative to a fixed observer on the riverbank. The change in velocity and

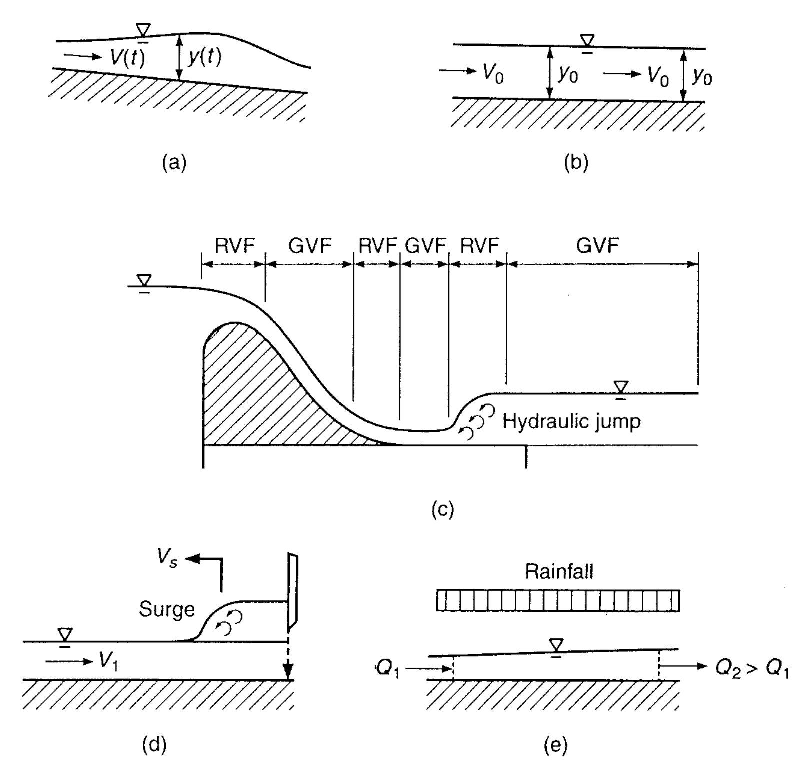

FIGURE 1.1
Types of open channel flow: **(a)** unsteady; **(b)** steady, uniform; **(c)** steady, gradually varied (GVF) and steady, rapidly varied (RVF); **(d)** unsteady, rapidly varied; **(e)** spatially varied.

depth in a large river may occur so gradually and over such long distances that the observer can see only a gradual rise and fall of river stage. If the flood wave results from a dam break, on the other hand, an abrupt change in depth and velocity and a distinct wave front or surge may be observed. In the former case, only near the peak of the flood wave could the flow be considered approximately steady, or quasi-steady, allowing steady flow analyses.

Spatial variations in velocity and depth in the flow direction are distinguished by the terms *uniform* and *nonuniform*. In a uniform flow, the mean cross-sectional velocity and depth are constant in the flow direction, as shown in Figure 1.1b. This flow condition is difficult to create in the laboratory and rarely occurs in the field, but often is used as the basis for open channel design. It requires the existence of a channel of uniform geometry and slope in the flow direction; that is, a prismatic channel. The nonuniform flow condition can be divided into two types: gradually varied and rapidly varied. Gradually varied flow is nonuniform flow, but the curvature of the free surface and of the accompanying streamlines is so slight that the transverse pressure distribution at any station along the flow can be approximated as hydrostatic. This assumption allows the flow to be treated with one-dimensional forms of the governing differential equations in which we are concerned with variation of the flow variables in the flow direction only. Fortunately, most river flows can be treated in this manner. Rapidly varied flow, on the other hand, is not amenable to this approach and often requires application of the momentum equation in control volume form as in the hydraulic jump or a two-dimensional formulation of the governing differential equations as in the highly curvilinear flow over a spillway crest. Examples of gradually varied and rapidly varied flow are shown in Figures 1.1c and 1.1d.

Spatially varied flow really is a class of nonuniform flow but owes its nonuniformity to variation in the flow discharge in the direction of motion as well as to an imbalance of gravity and resisting forces. Examples of spatially varied flow include side channel spillways and continuous rainfall additions to gutter flow, as shown in Figure 1.1e.

1.7 BASIC EQUATIONS

The basic equations of fluid mechanics are applied to open channel flow with some modifications due to the free surface. These equations are the continuity, momentum, and energy equations, which can be derived directly from the Reynolds transport theorem applied to a fixed control volume as shown in Figure 1.2a. The Reynolds transport theorem is derived in many elementary fluid mechanics textbooks (Roberson and Crowe 1997; White 1999) and is given by

$$\frac{dB}{dt} = \frac{d}{dt}\int_{cv} b\rho \, d\forall + \int_{cs} b\rho(\mathbf{V}\cdot\mathbf{n})\, dA \tag{1.2}$$

in which B = system property; t = time; b = the intensive value of B per unit mass m, dB/dm; ρ = fluid density; $\forall$ = volume of the control volume (cv); $\mathbf{V}$ = veloc-

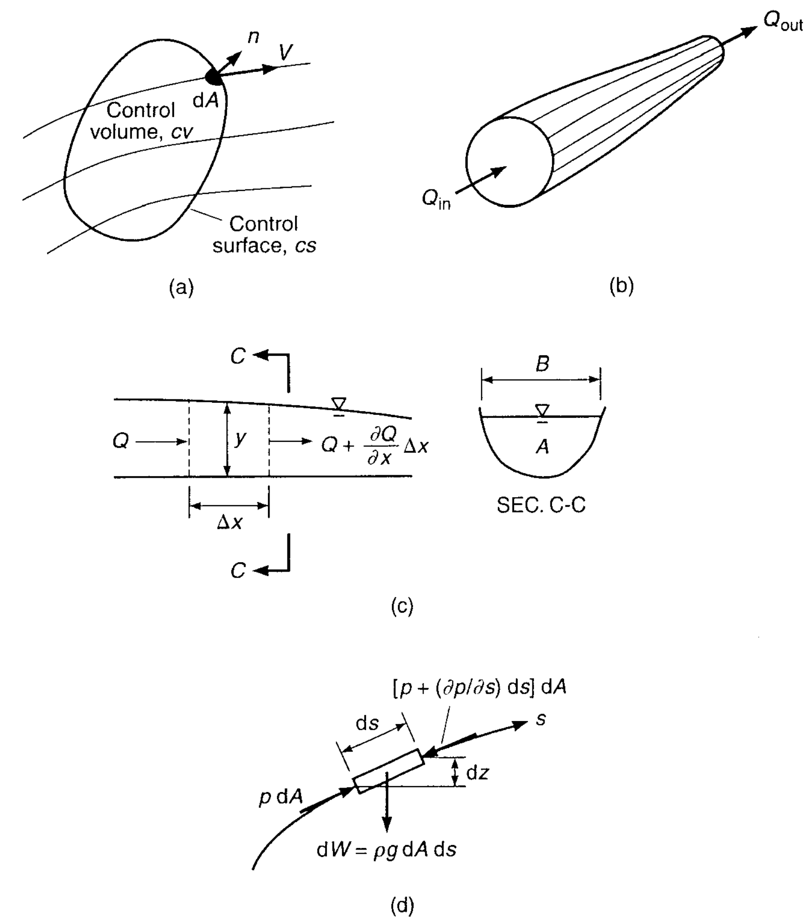

FIGURE 1.2
Control volumes **(a)** arbitrary control volume; **(b)** streamtube; **(c)** river reach; **(d)** streamline.

ity vector; $\mathbf{n}$ = outward normal unit vector; and A = area of the control surface (cs). The volume integral on the right hand side of Equation 1.2 sums up the values of the property per unit mass b over each mass element given by $\rho d\forall$. In the surface integral in Equation 1.2, $(\rho\mathbf{V}\cdot\mathbf{n})\, dA$ represents the mass flux through an elemental area dA on the control surface. The dot product of the velocity vector with the unit outward normal $(\mathbf{V}\cdot\mathbf{n})$ determines the component of the velocity perpendicular to the surface since only that component can carry the property through the surface. Furthermore, the dot product is positive for outward fluxes and negative for inward fluxes into the control volume. Thus, the surface integral sums up the products of the property per unit mass b and the mass flux over the control surface to give the net outward flux of the property. In summary, Equation 1.2 states that the time rate of change of the system property is the sum of the time rate of change of the property inside the control volume and the net outward flux of the property through the control surface.

The Reynolds transport theorem can be applied to the properties of mass, momentum, and energy to obtain the control volume form of the corresponding governing conservation equations. The control volume forms of the equations can be simplified for the case of steady, one-dimensional flow and used in the analysis of many open channel flow problems.

In the case of mass m, the property $B = m$ and it follows that $\mathrm{d}B/\mathrm{d}t = 0$ and $b = \mathrm{d}B/\mathrm{d}m = 1$, so that

$$0 = \frac{\mathrm{d}}{\mathrm{d}t}\int_{cv} \rho \mathrm{d}\forall + \int_{cs} \rho(\mathbf{V}\cdot\mathbf{n})\,\mathrm{d}A \tag{1.3}$$

which means simply that the time rate of change of mass inside the control volume in the first term must be balanced by the net outward mass flux through the control surface expressed by the second term. Now, in the case of steady flow of an incompressible fluid for the one-dimensional streamtube shown in Figure 1.2b, we have the familiar form of the continuity equation:

$$\int_{cs} (\mathbf{V}\cdot\mathbf{n})\,\mathrm{d}A = 0 = \Sigma Q_{\text{out}} - \Sigma Q_{\text{in}} \tag{1.4}$$

in which ΣQ = summation of the volume fluxes in or out of the control volume. The mean cross-sectional velocity, V_s, is defined as the volume flux divided by the cross-sectional area of flow perpendicular to the streamlines such that the volume flux can be written as

$$Q = \int_{cs} v_s\,\mathrm{d}A = V_s A \tag{1.5}$$

in which v_s is the point velocity in the streamline direction; V_s is the mean cross-sectional velocity; and A is the cross-sectional area of flow.

Equation 1.3 also can be written in differential form for the general case of unsteady open channel flow of an incompressible fluid. If the control volume is considered to have a differential length Δx, as shown in Figure 1.2c, then as Δx approaches zero, Equation 1.3 becomes

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1.6}$$

At any cross section, the time rate of change of flow area due to unsteadiness as the free surface rises or falls must be balanced by a spatial gradient in the volume flux Q in the flow direction. For steady flow, $\partial A/\partial t$ is zero by definition and $\partial Q/\partial x$ then also must become zero, which implies that the volume flux Q is constant along the channel, in agreement with Equation 1.4. The differential form of the continuity

equation as given by Equation 1.6 will be applied in the numerical analysis of unsteady open channel flow in Chapter 8.

If we turn now to the property of momentum, the fundamental property B in the Reynolds transport theorem becomes a vector quantity defined by the linear momentum $\mathbf{B} = m\mathbf{V}$, in which m = mass and $\mathbf{V}$ = velocity vector. The total derivative $\mathrm{d}\mathbf{B}/\mathrm{d}t$ is exactly the vector sum of forces $\Sigma\mathbf{F}$ acting on the control volume according to Newton's second law. In this case, $\mathrm{d}\mathbf{B}/\mathrm{d}m = \mathbf{V}$ and the Reynolds transport theorem for a fixed control volume becomes a vector equation, which can be written as

$$\Sigma\mathbf{F} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{cv} \mathbf{V}\rho\,\mathrm{d}\forall + \int_{cs} \mathbf{V}\rho(\mathbf{V}\cdot\mathbf{n})\,\mathrm{d}A \tag{1.7}$$

Equation 1.7 states that the vector sum of forces acting on the control volume is equal to the time rate of change of linear momentum inside the control volume plus the net momentum flux out of the control volume through the control surface. In fact, this equation can be thought of simply as Newton's second law applied to a fluid. It is crucial to note that Equation 1.7 is a vector equation that represents three separate equations, written in each coordinate direction with the appropriate components of each vector quantity.

For the special case of the streamtube control volume in Figure 1.2b, the steady, one-dimensional form of the momentum equation in the stream direction, s, is given by

$$\Sigma F_s = \int_{cs} \rho v_s(\mathbf{V}\cdot\mathbf{n})\,\mathrm{d}A = \Sigma(\beta\rho Q V_s)_{\text{out}} - \Sigma(\beta\rho Q V_s)_{\text{in}} \tag{1.8}$$

in which v_s is the point velocity in the streamtube direction; V_s is the mean velocity; and β is the momentum flux correction coefficient to account for a nonuniform velocity distribution. The momentum equation as given by Equation 1.8 states that the vector sum of external forces in the streamtube direction is equal to the momentum flux out of the control volume in the s direction minus the momentum flux into the control volume in the s direction.

The momentum flux correction coefficient β in Equation 1.8 is defined by

$$\beta = \frac{\int_A v_s^2\,\mathrm{d}A}{V_s^2 A} \tag{1.9}$$

to correct for the substitution of the mean velocity squared for the point velocity squared and bringing it outside the integral in Equation 1.8. In turbulent flow in prismatic channels, the value of β is not significantly greater than the value of unity, which is the value for a uniform velocity distribution. In other open channel flow situations such as immediately downstream of a bridge pier, or in a river channel with floodplain flow, the value of β cannot be taken as unity because of the highly nonuniform velocity distributions in these situations.

It is important to note that the volume flux, Q, has been substituted for AV_s in Equation 1.8 and that the remaining V_s in the momentum flux term is the component of mean velocity in the direction in which the forces are summed. The outward volume flux takes a positive sign from $(\mathbf{V} \cdot \mathbf{n})$ because of the positive outward unit vector, and a negative sign goes with the inward volume flux for the same reason. The sign of V_s depends on the chosen positive direction for the force summation. If the forces are being summed in a direction x that is different from the streamtube direction, the volume flux remains unchanged but the component velocity is taken in the x direction with the appropriate sign. In the x direction, Equation 1.8 becomes

$$\Sigma F_x = \Sigma(\beta\rho Q V_x)_{\text{out}} - \Sigma(\beta\rho Q V_x)_{\text{in}} \tag{1.10}$$

If the momentum equation is applied to a differential control volume along a streamline, as in Figure 1.2d, and only pressure and gravity forces are considered, the result is Euler's equation for an incompressible, frictionless fluid:

$$-\frac{\partial p}{\partial s} - \rho g \frac{\partial z}{\partial s} = \rho \frac{\partial v_s}{\partial t} + \rho v_s \frac{\partial v_s}{\partial s} \tag{1.11}$$

in which p = pressure; z = elevation; v_s = streamline velocity; t = time; and s = coordinate in the streamline direction. If only steady flow is considered and Euler's equation is integrated along a streamline, the result is the familiar Bernoulli equation written here in terms of head between any two points along the streamline:

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} \tag{1.12}$$

in which γ is the specific weight of water $= \rho g$. In this form, the Bernoulli equation terms have dimensions of energy or work per unit weight of fluid, and so it is truly a work-energy equation derived from, but independent of, the momentum equation. The terms are scalars and represent pressure work, potential energy, and kinetic energy in that order. For applications to open channel flow, we need to expand the equation from a streamline to a streamtube and include the energy head loss term due to friction, h_f, for a real fluid, which results in

$$\frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_f \tag{1.13}$$

This expansion of the Bernoulli equation to a streamtube with head loss included is called the *extended Bernoulli equation* or the *energy equation.* It requires the assumption of a hydrostatic pressure distribution at points 1 and 2, because this means that the piezometric head $(p/\gamma + z)$ is a constant across the cross section. The use of the mean velocity in the velocity head term necessitates a kinetic energy flux correction coefficient defined by

$$\alpha = \frac{\int_A v_s^3 \, dA}{V_s^3 A} \tag{1.14}$$

to account for a nonuniform velocity distribution. As we shall see in succeeding chapters, the value of α can be significantly larger than unity in rivers with overbank flow and therefore cannot be neglected.

To emphasize the independence of the extended Bernoulli or energy equation from the momentum equation, it should be pointed out that the energy equation can be derived in a more general way from the Reynolds transport theorem and the first law of thermodynamics:

$$\frac{dE}{dt} = \frac{dQ_h}{dt} - \frac{dW_s}{dt} - \frac{dW_p}{dt} = \frac{d}{dt}\int_{cv} e\rho \, d\forall + \int_{cs} e\rho(\mathbf{V}\cdot\mathbf{n}) \, dA \tag{1.15}$$

in which B has been replaced by the total energy E; Q_h = the heat transfer to the fluid; W_s = the shaft work done by the fluid on hydraulic machines; W_p = the work done by the fluid pressure forces; and e is dE/dm = the internal energy plus kinetic energy plus potential energy per unit mass. For steady, one-dimensional flow of an incompressible fluid, the energy balance given by Equation 1.15 reduces to Equation 1.13, in which the head loss term represents the irreversible change in internal energy and the energy converted into heat due to viscous dissipation (White 1999).

The continuity equation is a statement of the conservation of mass. Likewise, the energy equation expresses conservation of energy. It is a scalar equation and in the form of work/energy because of the spatial integration of $\Sigma\mathbf{F} = m\mathbf{a}$. The momentum equation also comes from Newton's second law applied to a fluid but is a vector equation that states that the sum of forces in any coordinate direction is equal to the change in momentum flux in that direction. In the control volume form, the momentum equation can be applied to quite complicated flow situations, as long as the external forces on the control volume can be quantified. The energy equation, on the other hand, requires the capability of quantifying energy dissipation inside the control volume.

Often, all three fundamental equations are applied simultaneously to solve what otherwise would be intractable problems. The hydraulic jump is an example in which the momentum and continuity equations are applied first to obtain the sequent depth (depth after the jump), and then the energy equation is employed to solve for the unknown energy loss.

Even experienced hydraulicians sometimes misapply the momentum and energy equations. The cardinal rule is that the energy equation must include all significant energy losses and the momentum equation must include all significant forces. Breaking this rule sometimes leads to conflicting results from the momentum and energy equations because of misapplication rather than a breakdown of the fundamental physical laws.

1.8 SURFACE VS. FORM RESISTANCE

Flow resistance in fluid flow can result from two fundamentally different physical processes, which take on special meaning when we discuss open channel flow resistance coefficients. Surface resistance is the traditional form of resistance

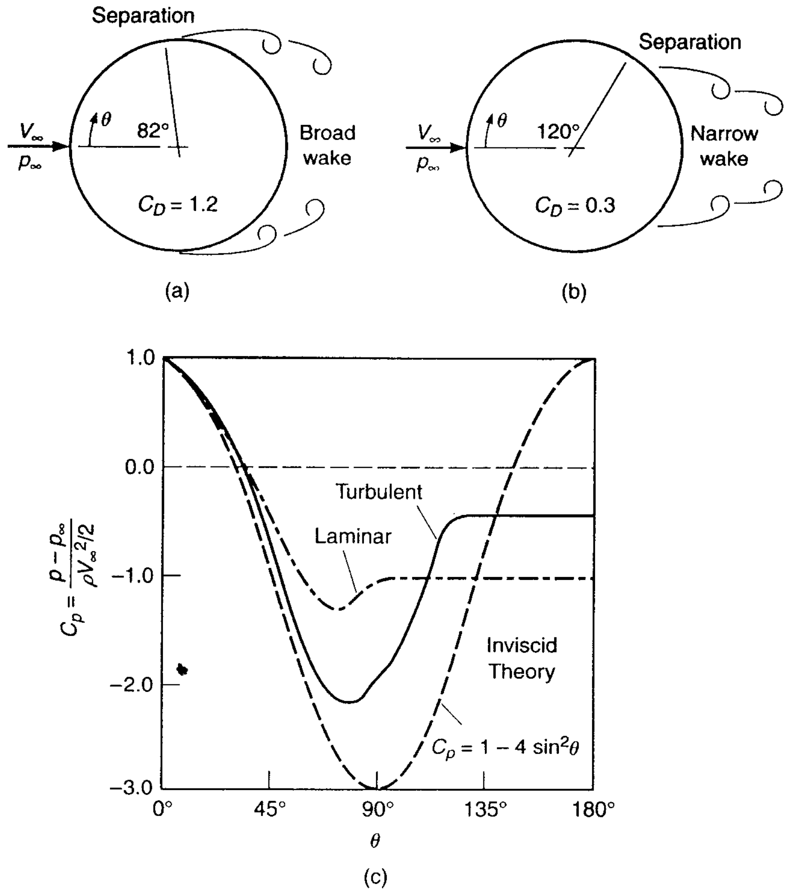

FIGURE 1.3
Separation and form resistance in real fluid flow around a circular cylinder: **(a)** laminar separation; **(b)** turbulent separation; **(c)** real and ideal fluid pressure distributions (White 1999). (*Source: F. White, Fluid Mechanics, 4e, © 1999, McGraw-Hill. Reproduced with permission of The McGraw-Hill Companies.*)

resulting from surface friction or shear stress at a solid boundary. Integration of the shear stress over the surface area of the circular cylinder in Figure 1.3, for example, would result in surface drag.

Surface resistance alone cannot account for the measured flow resistance of a blunt object, such as a circular cylinder. Because of the phenomenon of flow separation of a real fluid, an asymmetric pressure distribution occurs around the circular cylinder, leading to form drag as shown in Figure 1.3 with higher pressure on the upstream face of the cylinder than on the downstream face in the zone of separation. In contrast, inviscid flow theory predicts a symmetric pressure distribution and no form drag (as well as no surface drag) on the cylinder, as shown in Figure 1.3. If the component of the pressure force in the flow direction is obtained by integrating the real fluid pressure distribution around the sphere, the result is a form drag or form resistance that is completely separate from surface drag. The total drag then is the sum of the surface drag and form drag. The magnitude of the form drag depends highly on the point of separation, which is different in the laminar and turbulent cases, as shown by Figure 1.3. In open channel flow, the resistance offered by large roughness elements or alluvial bed forms may be due largely to form resistance. This point will be discussed in more detail in Chapters 4 and 10.

1.9 DIMENSIONAL ANALYSIS

The purpose of dimensional analysis is to reduce the number of independent variables in an open channel flow problem or any other fluid mechanics problem by transforming the dependent variable and several independent variables that form a functional relationship into a smaller number of dimensionless ratios. This reduces the number of experiments involved in developing an experimental relationship, since only the independent dimensionless parameters need to be varied rather than each individual independent variable. Rather than varying the velocity, depth, and gravitational acceleration independently in a hydraulic jump experiment, for example, it is necessary to vary only the Froude number, which is a dimensionless combination of these variables, and present the results for the ratio of depths before and after the jump in terms of the Froude number. In addition, the dimensionless variables often represent ratios of forces, such as inertia and gravity, so that the magnitude of a particular dimensionless variable and its variation in a given experiment relate to an understanding of the physics of the flow situation. Furthermore, presentation of experimental results in terms of dimensionless variables generalizes the results to a wider range of applications and confirms the validity of the dimensionless ratios chosen to model a particular flow phenomenon.

If the governing equations can be completely formulated for a given problem, the equations can be nondimensionalized to deduce the embedded dimensionless parameters of importance. For example, application of the momentum equation to a hydraulic jump and nondimensionalization of the resulting equation for the depth after the jump results directly in the appearance of the Froude number as the only independent dimensionless parameter for this problem. The necessary condition for nondimensionalization of an equation is dimensional homogeneity, which simply requires every term to have the same dimensions in any properly posed equation describing a physical phenomenon. Once the governing equations are transformed into dimensionless form, the solution can be obtained in terms of the resulting dimensionless variables, either analytically or numerically, for a completely general solution. This solution can be applied to similar flow situations under conditions different from those for which the results were obtained, so long as the ranges of the dimensionless variables are the same.

In some cases, equations of open channel flow such as the Manning's equation or the head-discharge equation for flow over a weir at first may not appear to be dimensionally homogeneous. In these cases, some "constant" must have dimensions for the equation to be dimensionally homogeneous. If the equation for discharge Q over a sharp-crested weir, for example, is written as a constant C_1 times $LH^{3/2}$, where L is the crest length and H is the head on the crest, it is clear that the equation is not dimensionally homogeneous unless C_1 has dimensions of length to the 1/2 power divided by time. These in fact are the dimensions of the square root of the gravitational acceleration, g, which has been incorporated implicitly into the value of C_1. This practice requires that the coefficient C_1 take on a different numerical value for different systems of units, which is less desirable than leaving the original equation in terms of the gravitational acceleration.

As an example of nondimensionalization of the governing equations, the inviscid flow solution shown in Figure 1.3 can be obtained from an application of Bernoulli's equation between the approach flow (variables with a subscript of ∞) and any point on the circumference of the cylinder:

$$p_\infty + \rho\frac{V_\infty^2}{2} = p + \rho\frac{v^2}{2} \tag{1.16}$$

If the equation is nondimensionalized, there results

$$\frac{p - p_\infty}{\rho\dfrac{V_\infty^2}{2}} = C_p = 1 - \left(\frac{v}{V_\infty}\right)^2 \tag{1.17}$$

in which C_p is defined as a dimensionless pressure coefficient. The solution for the pressure coefficient is obtained by substituting the inviscid flow solution for the circumferential velocity $v = 2V_\infty \sin\theta$ into Equation 1.17 with the result

$$C_p = 1 - 4\sin^2\theta \tag{1.18}$$

Equation 1.18 gives the theoretical distribution of the dimensionless pressure coefficient C_p shown in Figure 1.3. Thus, if the governing equation of a fluid mechanics problem is known, then the equation itself can be made dimensionless, as in Equation 1.17, and the resulting solution also will be dimensionless.

In many problems of open channel flow, the theoretical solution is not directly applicable without the addition of experimental results to evaluate unknown parameters, or it may not be possible to formulate and solve the governing equations in very complicated flows. This requires a different approach for obtaining the important dimensionless parameters of the problem. In the case of drag on a circular bridge pier, for example, specification of the experimental drag coefficient is necessary to calculate the drag force, which includes both surface and form drag, the latter of which is not easily calculated from the governing equations. Presentation of the experimental results for the drag force in dimensionless form requires a general technique such as that afforded by the Buckingham Π theorem (see, for example, White 1999). The Buckingham Π theorem can be stated as follows:

If a physical process involves a functional relationship among n variables, which can be expressed in terms of m basic dimensions, it can be reduced to a relation between $(n - m)$ dimensionless variables, or Π terms, by choosing m repeating variables, each of which is combined in turn with the remaining variables to form the Π terms as products of the variables taken to the appropriate powers. The m repeating variables must contain among them all basic dimensions found in all the variables but cannot themselves form a Π term.

In mathematical terms, if a dependent variable A_1 can be expressed in terms of $(n - 1)$ independent variables as

$$A_1 = f(A_2, A_3, \ldots, A_n) \tag{1.19}$$

then the Buckingham Π theorem allows the n variables to be expressed as a functional relation among $(n - m)$ Π groups:

$$\phi(\Pi_1, \Pi_2, \ldots, \Pi_{n-m}) = 0 \tag{1.20}$$

The basic dimensions usually are taken as mass (M), length (L), and time (T), although force (F), length, and time are an equally valid choice. The force dimension is uniquely related to the remaining dimensions by Newton's second law; that is, $F = MLT^{-2}$. In certain instances, the fundamental dimensions may be fewer than three; for example, only length and time may be involved. When choosing repeating variables, it is important to recognize that it is better not to choose the dependent variable as a repeating variable, so that it will appear in only one Π term.

If, for example, $n = 5$ and $m = 3$ with M, L, and T as the basic dimensions, the two Π terms can be found from

$$[\Pi_1] = M^0L^0T^0 = [A_2]^{x_1}[A_3]^{y_1}[A_4]^{z_1}[A_1] \tag{1.21}$$

$$[\Pi_2] = M^0L^0T^0 = [A_2]^{x_2}[A_3]^{y_2}[A_4]^{z_2}[A_5] \tag{1.22}$$

in which the square brackets denote "dimensions of" the enclosed variables; and A_2, A_3, and A_4 have been chosen as repeating variables. By substituting the dimensions of the variables into the right hand sides of Equations 1.21 and 1.22 and equating the exponents on M, L, and T on both sides of the equations, the resulting algebraic equations can be solved for the unknown exponents and the resulting Π terms.

Now consider the drag problem for a completely immersed cylinder in which the drag force, D, can be expressed in terms of the cylinder diameter, d; the cylinder length, l_c; the approach velocity, V_∞; the fluid density, ρ; and the fluid viscosity, μ:

$$D = f_1(d, l_c, V_\infty, \rho, \mu) \tag{1.23}$$

A total of six variables with all three basic dimensions (M, L, T) are represented, so there will be three Π terms. The repeating variables are chosen to be the density, velocity, and cylinder diameter, which contain among them M, L, and T as basic dimensions but do not themselves form a dimensionless group. The cylinder diameter and length could not be chosen together as repeating variables because they would form a Π group. First, the drag force is combined with powers of the repeating variables, either algebraically or by inspection, to yield the first Π term; then the same process is repeated for the cylinder length and the fluid viscosity. The result is given by

$$\frac{D}{\rho d^2 V_\infty^2} = f_2\left(\frac{l_c}{d}, \mathbf{Re}\right) \tag{1.24}$$

which gives the dimensionless drag ratio in terms of the Reynolds number, $\mathbf{Re} = \rho V_\infty d/\mu$ and the ratio of cylinder length to diameter, l_c/d. Traditionally, the drag ratio is redefined as a more general drag coefficient, applicable to other shapes of immersed objects as $D/(\rho A V_\infty^2/2)$, with A in the coefficient of drag defined as the frontal area of the immersed object projected onto a plane perpendicular to the oncoming flow ($l_c \times d$). Also, a factor of 2 is added to the definition of the drag coefficient as a matter of tradition. For an infinitely long cylinder, the ratio l_c/d no longer has an influence because there are no end effects, so the experimental coefficient of drag is determined from the Reynolds number alone and used to calculate the drag force.

The choice of the repeating variables is not unique, so there are equally valid alternative forms of the Π groups. If, for example, the repeating variables were chosen to be μ, V_∞ and d in the cylinder drag problem, the result would be

$$\frac{D}{\mu V_\infty d} = f_3\left(\mathbf{Re}, \frac{l_c}{d}\right) \tag{1.25}$$

However, the alternate dependent Π group in (1.25) could be deduced from taking the product of the drag ratio and Reynolds number in (1.24). In the same manner, the justification for replacing d^2 in the denominator of the drag ratio in (1.24) with the frontal area is that the drag ratio in (1.24) can be divided by l_c/d and replaced by the result. In general, it is possible to state that a new Π group can be formed as

$$\Pi_1' = \Pi_1^a \Pi_2^b \Pi_3^c \tag{1.26}$$

and used to replace one of the original Π groups.

In the more general case of several bridge piers, each with diameter d and spacing s between piers and in open channel flow with a finite depth of water y_0, the formation of gravity surface waves around the piers may give rise to additional flow resistance so that the drag force can be written as

$$D = f_4(d, s, y_0, V_\infty, \rho, \mu, g) \tag{1.27}$$

in which the gravitational acceleration has been added to the list of variables. Alternatively, the specific weight γ could be added to the list instead of g, but the ratio γ/ρ, which is equal to g, then would appear in the dimensionless group related to the gravity force. Now, there are eight variables and still three basic dimensions resulting in five Π groups that can be expressed as

$$\frac{D}{\rho\, d y_0 V_\infty^2} = f_5\left(\frac{d}{s}, \frac{d}{y_0}, \mathbf{Re}, \mathbf{F}\right) \tag{1.28}$$

The additional geometric variable results in an additional geometric ratio, and the introduction of the gravitational force necessarily brings into play the Froude number, **F**. The relative importance of the Π groups on the right hand side of (1.28) would be determined by experiments.

The existence of the free surface in open channel flow inevitably involves the gravity force, either through the formation of surface waves, the existence of a component of the body force in the flow direction, or a differential pressure force due to changes in depth. Therefore, a dimensional analysis of an open channel flow problem includes the gravitational acceleration in the list of variables, and the Froude number necessarily emerges as an important dimensionless parameter, as discussed previously.

The choice of independent and dependent variables is crucial to the success of dimensional analysis. There can be only one dependent variable, and the independent variables must not be redundant; that is, one of the independent variables cannot be obtained from some combination of the others. The inclusion of extra independent variables that are truly independent is not fatal because the experimental results will show which of the resulting dimensionless groups is unimportant, but failing to include a significant independent variable can give an incomplete experimental relationship. Ultimately, such decisions are made in the course of research on a particular problem and may involve trial and error to arrive at the final set of important dimensionless ratios.

1.10 COMPUTER PROGRAMS

Some computer programs are given in Appendix B in Visual BASIC code, which is applicable to the Microsoft Windows environment. The BASIC language has evolved from a DOS-based language to the present form that utilizes the graphical user interface of Windows. It is an event-driven language composed of both form modules, which contain the graphical user interface, and standard modules, which contain the computational code. The programs in the appendix include standard modules that consist of numerical procedures or subprograms. They can be converted easily to other languages such as Fortran or C, combined with form modules in Visual BASIC for input and output, or incorporated into Excel spreadsheets using Visual BASIC for Applications. The purpose here is to develop the core methodology for the use of numerical analysis to solve open channel flow problems. To this end, Appendix A contains some basic material on numerical methods that will be used throughout the text. Appendix B includes some example programs that are intended to serve as learning tools to explore the application of numerical techniques to open channel flow problems.

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EXERCISES

1.1. Classify each of the following flows as steady or unsteady from the viewpoint of the observer:

Flow	*Observer*
(*a*) Flow of river around bridge piers.	(1) Standing on bridge.
	(2) In boat, drifting.
(*b*) Movement of flood surge downstream.	(1) Standing on bank.
	(2) Moving with surge.

1.2. At the crest of an ogee spillway, as shown in Figure 1.1c, would you expect the pressure on the face of the spillway to be greater than, less than, or equal to the hydrostatic value? Explain your answer.

1.3. The river flow at an upstream gauging station is measured to be 1500 m^3/s, and at another gauging station 3 km downstream, the discharge is measured to be 750 m^3/s at the same instant of time. If the river channel is uniform, with a width of 300 m, estimate the rate of change in the water surface elevation in meters per hour. Is it rising or falling?

1.4. A paved parking lot section has a uniform slope over a length of 100 m (in the flow direction) from the point of a drainage area divide to the inlet grate, which extends across the lot width of 30 m. Rainfall is occurring at a uniform intensity of 10 cm/hr. If the detention storage on the paved section is increasing at the rate of 60 m^3/hr, what is the runoff rate into the inlet grate?

1.5. A rectangular channel 6 m wide with a depth of flow of 3 m has a mean velocity of 1.5 m/s. The channel undergoes a smooth, gradual contraction to a width of 4.5 m.
(*a*) Calculate the depth and velocity in the contracted section.
(*b*) Calculate the net fluid force on the walls and floor of the contraction in the flow direction.
In each case, identify any assumptions that you make.

1.6. A bridge has cylindrical piers 1 m in diameter and spaced 15 m apart. Downstream of the bridge where the flow disturbance from the piers is no longer present, the flow depth is 2.9 m and the mean velocity is 2.5 m/s.
(*a*) Calculate the depth of flow upstream of the bridge assuming that the pier coefficient of drag is 1.2.
(*b*) Determine the head loss caused by the piers.

1.7. A symmetric compound channel in overbank flow has a main channel with a bottom width of 30 m, side slopes of 1:1, and a flow depth of 3 m. The floodplains on either side of the main channel are 300 m wide and flowing at a depth of 0.5 m. The mean velocity in the main channel is 1.5 m/s, while the floodplain flow has a mean velocity of 0.3 m/s. Assuming that the velocity variation within the main channel and the floodplain subsections is much smaller than the change in mean velocities between subsections, find the value of the kinetic energy correction coefficient α.

1.8. The power law velocity distribution for fully rough, turbulent flow in an open channel is given by

$$\frac{u}{u_*} = a\left(\frac{z}{k_s}\right)^{1/6}$$

in which u = point velocity at a distance z from the bed; u_* = shear velocity = $(\tau_0/\rho)^{1/2}$; τ_0 = bed shear stress; ρ = fluid density; k_s = equivalent sand grain roughness height; and a = constant.
(*a*) Find the ratio of the maximum velocity, which occurs at the free surface where z = the depth, y_0, to the mean velocity for a very wide channel.
(*b*) Calculate the values of the kinetic energy correction coefficient α and the momentum flux correction coefficient β for a very wide channel.

1.9. An alternative expression for the velocity distribution in fully rough, turbulent flow is given by the logarithmic distribution

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right)$$

in which κ = the von Karman constant = 0.40; $z_0 = k_s/30$; and the other variables are the same as defined in Exercise 1.8. Show that α and β for this distribution in a very wide channel are given by

$$\alpha = 1 + 3\varepsilon^2 - 2\varepsilon^3$$

$$\beta = 1 + \varepsilon^2$$

in which $\varepsilon = (u_{max}/V) - 1$; u_{max} = maximum velocity; and V = mean velocity.

1.10. In a hydraulic jump in a rectangular channel of width b, the depth after the jump y_2 is known to depend on the following variables:

$$y_2 = f[y_1, q, g]$$

in which y_1 = depth before the jump; q = discharge per unit width = Q/b; and g = gravitational acceleration. Complete the dimensional analysis of the problem.

1.11. The backwater Δy caused by bridge piers in a bridge opening is thought to depend on the pier diameter and spacing, d and s, respectively; downstream depth, y_0; downstream velocity, V; fluid density, ρ; fluid viscosity, μ; and gravitational acceleration, g. Complete the dimensional analysis of the problem.